Section 2.1a Notes (see slides in same section)

- **Nuclear chart, decay modes, neutron- and proton driplines**

  When one examines the nuclear chart, one finds that a given element, characterized by the proton number Z, has almost always several stable nuclear isotopes which are characterized by the variable neutron number N. For example, the element oxygen, $Z = 8$, has 3 stable isotopes with neutron numbers $N = 8, 9, 10$. In addition, there are 12 known unstable oxygen isotopes. Another example is the element tin, $Z = 50$, which has 10 stable isotopes and 28 known unstable isotopes.

  One of the fundamental questions of nuclear structure physics is: what are the limits of nuclear stability? How many neutrons can we add to a given nuclear isotope before it becomes unstable against spontaneous neutron emission (neutron radioactivity)? If one connects the isotopes with zero neutron separation energy, $S_n = 0$, in the nuclear chart one obtains the neutron dripline. Similarly, the proton dripline is defined by the condition $S_p = 0$. There are less than 300 stable nuclear isotopes to be found in nature. However, an additional 2700 isotopes have been created in experiments, most of these in reactions using low-energy heavy-ion accelerators. Nuclei in between the proton and neutron driplines are unstable against beta-decay. Heavier nuclei also decay by α-particle emission or by spontaneous fission. Spontaneous fission occurs for nuclei with large proton numbers (actinide elements with $Z = 89 - 103$ and transactinides with $Z \geq 104$). Nuclei outside the driplines decay by spontaneous neutron emission or proton radioactivity. Measured half lives for nuclear isotopes vary between billions of years (near the stability line) to microseconds for the most unstable nuclei.

- **Radioactive decay law: decay rate, half-life, mean life, and level width**

  Reference: K. Heyde, "Basic Ideas and Concepts in Nuclear Physics", 2nd ed.

  Suppose that at time t_0 there are $N_0 = N(t_0)$ nuclei present. As a result of some radioactive decay (α, β, γ-decay, or nuclear fission), the number of original nuclei left at time $t > t_0$ is given by the **radioactive decay law**

$$N(t) = N_0 \; e^{-\lambda t} \; .$$

  The quantity λ is called the **radioactive decay rate**. This law was first formulated by Ernest Rutherford and is now experimentally well-established.

  The reciprocal of the decay rate is called the **mean life** τ

$$\tau = \frac{1}{\lambda}$$

  with the property $N(\tau) = N_0 \; e^{-1}$, i.e. after one mean life has passed, the number of original nuclei has dropped to a fraction of $1/e$ of the original value. We define the **half-life** $T_{1/2}$ by the condition

$$N(T_{1/2}) = N_0/2$$

  which yields

$$N_0 \; e^{-\lambda T_{1/2}} = N_0/2 \qquad \rightarrow \; e^{-\lambda T_{1/2}} = 1/2 \; .$$

  Taking the natural logarithm of this equation one finds the connection between half-life, decay rate, and mean life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \; \ln 2 \approx 0.693 \; \tau \; .$$

If we plot the relative number of nuclei at time t on a logarithmic scale

$$\ln(\frac{N(t)}{N_0}) = \ln(e^{-\lambda t}) = -\lambda t$$

we obtain a **straight line whose slope determines the decay rate** and hence the half-life. This method is useful for determining relatively short half-lives of nuclei.

On the other hand, if the half-life is very long (e.g. $T_{1/2} = 4.47$ billion years for α- decay of ^{238}U) one must obtain the half-life by measuring the **activity** $A(t)$ of the radioactive substance which is defined as the number of decays per unit time

$$A(t) = |dN(t)/dt| = \lambda N_0 \; e^{-\lambda t} = A_0 e^{-\lambda t} \; .$$

The activity can be measured in an electronic counter experiment. In this case, the decay rate can be obtained by plotting the relative activity on a logarithmic scale versus time

$$\ln(\frac{A(t)}{A_0}) = -\lambda t \; .$$

As a result of the radioactive decay, the system has a finite lifetime. From Heisenberg's uncertainty relation for energy and time $\Delta E \cdot \Delta t \approx \hbar$ we can compute the **level width** Γ of the system

$$\Gamma = \Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar}{\tau} = \hbar\lambda \; .$$

It is possible to derive the radioactive decay law using time-dependent perturbation theory (see e.g. A.S. Davydov, Quantum Mechanics, §80). In first-order perturbation theory, the decay rate is given by **Fermi's Golden Rule**

$$\lambda_{i\to f} = \frac{2\pi}{\hbar} |< f|H_{int}|i >|^2 \frac{dn}{dE} \; .$$

From this expression we conclude that the decay rate depends on 1) a transition matrix element involving the wave functions of the initial and final states of the nucleus and an interaction Hamiltonian, and 2) on the density of final states dn/dE for the particular process which must be computed by phase space considerations involving energy and momentum conservation of the particles (for details see Heyde's textbook, 2nd edition, ch. 3.2). The transition matrix elements depend on the particular radioactive process. In the case of γ-decay, for example, one finds that these are proportional to the electric multipole matrix elements

$$< f|H_{int}|i > \propto < f|Q_{\ell m}|i > \; .$$

A detailed discussion of EM multipole radiation will be given in Section 3.1a. If you are interested in the theory of β-decay, you may wish to consult Heyde's textbook, 2nd edition, ch. 5.

- **α-decay and spontaneous fission: tunneling through a potential barrier**

  The measured half-lives for α-decay and spontaneous fission vary by many orders of magnitude, ranging from billions of years to fractions of a second. This can be explained by the fact that both processes involve tunneling through a potential barrier, and the barrier penetrability depends exponentially on the height and with of the potential barrier.

Already in 1928, George Gamov explained spontaneous α-particle emission by barrier tunneling based on the semiclassical WKB method. The potential barrier is formed by a superposition of the repulsive Coulomb potential and the attractive nuclear potential between the α-particle and the daughter nucleus. The α-decay rate is a product of three factors:

$$\lambda_\alpha = P_\alpha \cdot R_b \cdot D \ ,$$

where P_α denotes the probability that an α-particle is formed in the nuclear surface region ($P_\alpha \approx 1$ in heavy nuclei). The quantity R_b is the rate of "barrier assaults" per unit time which can be estimated from the velocity of the α-particle and the width of the potential well, and the quantity D is the WKB barrier transmission coefficient (see e.g. Shankar, QM, 2nd edition, p.444)

$$D = \exp\left(-\frac{2}{\hbar}\int_{x_1}^{x_2} dx\sqrt{2m[V(x)-E]}\right) \ .$$

Gamov approximated the potential $V(x)$ by using a point-Coulomb potential which abruptly drops below zero at $x = R$, where $R = 1.2fm \times A^{1/3}$ is the nuclear radius. Details of the calculations and resulting half-lives for α-decay can be found in Heyde's textbook, 2nd edition, ch. 4.4.

The theoretical description of spontaneous fission is substantially more complicated. Most actinide nuclei involve a double-humped fission barrier, and in some of the thorium isotopes even a triple-humped barrier has been reported (see Section 3.2a). Qualitatively, these features can be understood by the phenomenological deformed shell model (see Sections 3.1c and 3.1d). Microscopic calculations of the fission potential based on the static Hartree-Fock-Bogoliubov theory (with added quadrupole and octupole constraints), are currently state-of-the art (see Section 4.6), but a full dynamical description of fission (perhaps based on time-dependent Hartree-Fock theory) is one of the grand challenges in the theory of large-amplitude collective motion.

- **Transuranium isotopes and the frontier of superheavy elements**

  By the mid-1950s the first transuranic elements were synthesized by bombarding heavy element targets with neutrons: Neptunium ($Z = 93$), Plutonium ($Z = 94$), Einsteinium ($Z = 99$), and Fermium ($Z = 100$). Other transuranic elements were created by using beams of alpha particles, giving rise to elements $Z = 95 - 98$ and $Z = 101$. Between 1958 and 1974, nuclear physicists created elements $Z = 102 - 106$ with light-ions beams ($Z \leq 5$). Starting in the mid 1970s, and continuing until present time, heavy-ion accelerators allowed the production of superheavy elements with $Z = 106 - 118$. The main goal of this research area is to reach the "superheavy island of stability". Its precise location is not known. Varies theories predict this island to occur near the proton numbers $Z = 114, 120, 126$ and magic neutron number $N = 184$.

- **Radioactive Ion Beam (RIB) physics: the frontier of the neutron dripline**

  The neutron-rich side of the nuclear chart exhibits thousands of nuclear isotopes still to be explored ("terra incognita"). Some of these exotic nuclei can be studied with existing first-generation Radioactive Ion Beam Facilities. Several countries are constructing new 'second generation' RIB facilities (RIKEN in Japan, FAIR in Germany, GANIL in France). In the United States, construction has begun of FRIB (Facility for Rare Isotope Beams) at Michigan State University.

Theories predict profound differences between the known isotopes near stability and the exotic nuclei at the driplines: for neutron-rich nuclei, as the Fermi level approaches the particle continuum at $E = 0$, weakly bound neutron states couple strongly to the continuum giving rise to neutron halos and neutron skins. Theories also expect large pairing correlations and new types of collective modes, a weakening of the spin-orbit force leading to a quenching of the shell gaps, and perhaps new magic numbers.

Furthermore, Radioactive Ion Beam Facilities will allow us to address fundamental questions in nuclear astrophysics: more than half of all elements heavier than iron are thought to be produced in supernovae explosions by the rapid neutron capture process (r-process). The r-process path goes through the "terra incognita" of neutron-rich isotopes which can only be studied with these new RIB facilities.