Part 4: Microscopic nuclear structure and reaction theories

4.1 Nucleon-Nucleon (NN) interaction

There are several approaches to obtain the NN interaction potential. Most important are these

a) construct V_{NN} from general invariance principles. We will see that this predicts the spin- and isospin-dependence, but not the radial dependance of the potential

b) from meson-exchange theories; the dominant term at large r is the "one-pion exchange potential" (OPEP).

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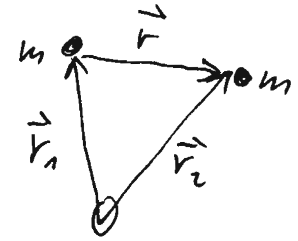
a) In 1941, Eisenbud and Wigner gave a very general discussion of the mathematical structure of V_{NN} based on invariance principles in physics. They assumed a potential of the form



$$V_{NN}(\underbrace{\vec{r}_1, \vec{r}_2}_{\text{position}}, \underbrace{\vec{p}_1, \vec{p}_2}_{\text{linear mom.}}, \underbrace{\vec{\sigma}_1, \vec{\sigma}_2}_{\text{spin}}, \underbrace{\vec{\tau}_1, \vec{\tau}_2}_{\text{isospin}}).$$

Symmetry requirements:

- translational inv: $\vec{r}_i \to \vec{r}_i + \underbrace{\vec{a}}_{\text{const. vector}}$

$\Rightarrow$ V_{NN} depends only on relative distance vector

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2$$

$$\Rightarrow \quad V_{NN} = V_{NN}(\vec{r}, \vec{p}_1, \vec{p}_2, \dots)$$

- Galilei inv: $\quad \vec{p}_i \to \vec{p}_i + \vec{P}_0$ const. vector

$\Rightarrow$ V_{NN} depends only on rel. momentum

$$\vec{p} = \vec{p}_1 - \vec{p}_2$$

$$\Rightarrow \boxed{V_{NN} = V_{NN}(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2, \vec{\tau}_1, \vec{\tau}_2)}$$

- approx. charge indep. of strong int. $\rightsquigarrow$ isospin scalar

experiments reveal that for the strong part of V_{NN}

$$V_{pp} \approx V_{nn} \approx V_{np}$$

$\Rightarrow$ invar. under rotation in isospin space $\rightsquigarrow$ isospin scalar. Possible scalars:

$$\left\{\begin{array}{l} c \text{ (const.)} \\ \vec{\tau}_i^{\,2} = \vec{\tau}_i \cdot \vec{\tau}_i \quad (i=1,2) \\ \quad = 3 \text{ (trivial constants)} \\ \vec{\tau}_1 \cdot \vec{\tau}_2 \end{array}\right\} \Rightarrow \left\{\begin{array}{c} 1 \\ \vec{\tau}_1 \cdot \vec{\tau}_2 \end{array}\right\}$$

Hence, V_{NN} has the form

$$\boxed{V_{NN} = v_1(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2) + v_2(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2}$$

- rotational inv:

The generators of rotations are the ang. momentum operators. Rot. inv. implies that all terms in $v_{1,2}(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2)$ above must be constructed to have a total ang. momentum of zero, i.e. they must be scalars in the combined (coordinate + spin)–space! There is a very large number of scalars that can be formed from $(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2)$, e.g.

$$\left.\begin{array}{l} \vec{r}\cdot\vec{r} = r^2, \quad \vec{\sigma}_1\cdot\vec{\sigma}_2, \quad \vec{\sigma}_i\cdot\vec{r}, \quad \vec{\sigma}_i\cdot\vec{p}, \quad \vec{r}\cdot\vec{p}, \\ \vec{\sigma}_i\cdot(\vec{r}\times\vec{p}), \quad \vec{r}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2), \quad \vec{p}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2), \text{ etc.} \end{array}\right\}$$

These possible combinations can be reduced further by 2 additional requirements:

- parity inv. (P):

$$\Rightarrow v(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2) \xrightarrow{P} v(-\vec{r}, -\vec{p}, \vec{\sigma}_1, \vec{\sigma}_2)$$

- time-reversal inv. (T):

$$\Rightarrow v(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2) \xrightarrow{T} v(\vec{r}, -\vec{p}, -\vec{\sigma}_1, -\vec{\sigma}_2)$$

note that the spins must transform like $\vec{l} = \vec{r}\times\vec{p}$!

This rules out the foll. combinations above:

$$\left\{\begin{array}{ll} \vec{\sigma}_i\cdot\vec{p} & (\text{not P-inv.}) \\ \vec{r}\cdot\vec{p} & (\text{not T-inv.}) \\ \vec{r}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2) & (\text{not P-inv.}) \\ \vec{p}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2) & (\text{violates } \underline{\text{both}} \text{ P and T}). \end{array}\right.$$

This leaves us with these building blocks for v_1, v_2:

$$v(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\sigma}_2) \propto \left\{\begin{array}{l} r^2 \rightarrow \text{arb. fctn } f(r) \\ \vec{\sigma}_1\cdot\vec{\sigma}_2 \\ \left.\begin{array}{l} \vec{\sigma}_i\cdot\vec{r} \\ \vec{\sigma}_i\cdot(\vec{r}\times\vec{p}) \end{array}\right\} (i = 1, 2) \end{array}\right.$$

The last term contains the possibility of a <u>strong spin-orbit interaction</u>, because

$$\boxed{\vec{L}\cdot\vec{S} = (\vec{r}\times\vec{p})\cdot\frac{\hbar}{2}(\vec{\sigma}_1+\vec{\sigma}_2)}$$

$\boxed{\text{Argonne } v_{18} \text{ NN-potential}}$

<u>Ref</u>: Wiringa, Stoks and Schiavalla, Phys. Rev. <u>C. 51</u>, 38 (1995)

This potential was developed by physicists at Argonne Nat. Lab. and it is composed of 18 terms – hence the name.

$$V_{NN}(\vec{r},\vec{p},\vec{\sigma}_i,\vec{\tau}_i) = \sum_{k=1}^{18} v_k(r)\, O_k$$

Note that one cannot obtain any information about the radial fctns $v_k(r)$ from invariance principles. One either needs to obtain these from meson exchange (see below) or from exp. N-N scattering data. Here are some of the important terms:

$$O_1 = 1 \quad (\text{"central component"} \mathrel{\hat{=}} c)$$
$$O_2 = \vec{\tau}_1\cdot\vec{\tau}_2 \quad (\text{"isospin component"} \mathrel{\hat{=}} \tau)$$
$$O_3 = \vec{\sigma}_1\cdot\vec{\sigma}_2 \quad (\text{"spin component"} \mathrel{\hat{=}} \sigma)$$
$$O_4 = (\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2) \quad (\text{"spin-isospin component"} \mathrel{\hat{=}} \sigma\tau).$$

The radial fctns corresponding to these 4 terms, i.e. $v_1 \dots v_4(r)$ are shown in fig. 6 of the above paper $\boxed{\Rightarrow \text{slide 2}}$

Another possible combination that can be constructed from the invariant blocks discussed above is the "tensor operator" S_{12} defined as

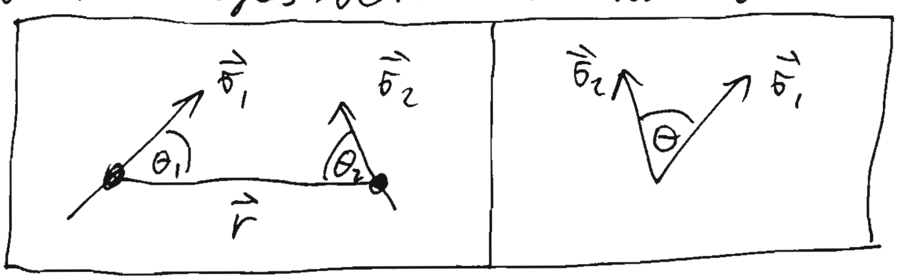
$$\boxed{S_{12} = 3\left(\vec{\sigma}_1 \cdot \frac{\vec{r}}{r}\right)\left(\vec{\sigma}_2 \cdot \frac{\vec{r}}{r}\right) - \vec{\sigma}_1 \cdot \vec{\sigma}_2}$$

Note that this operator is a second-rank tensor in either the spin-space or in conf. space alone, but a scalar in the combined conf. + spin space!

We observe that this tensor op. has the same form as the interaction pot. between 2 magnetic dipoles:

$$\boxed{V_{\text{magn. dipole}} \propto \frac{1}{r^3}\left[3(\vec{\mu}_1 \cdot \hat{\vec{r}})(\vec{\mu}_2 \cdot \hat{\vec{r}}) - \vec{\mu}_1 \cdot \vec{\mu}_2\right]}$$

A simple "geometric" interpretation of S_{12} is that it depends on the angle between the 2 spins $(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$ and on the angles between $\vec{r}$ and $\vec{\sigma}_i$:



One can show that the tensor force has a vanishing angular average, i.e. $\int d\Omega \, S_{12} = 0$.

2.0 6 (slides)

We already mentioned in the section ~~[illegible]~~ that the N-N interaction arises from virtual meson exchange

| discuss Slide #6 in this section |

and from the energy-time uncertainty relation $\Delta E \cdot \Delta t \sim \hbar$ we found that the range of the force, Δr, is of order of the Compton wavelength of the meson (λ_c):

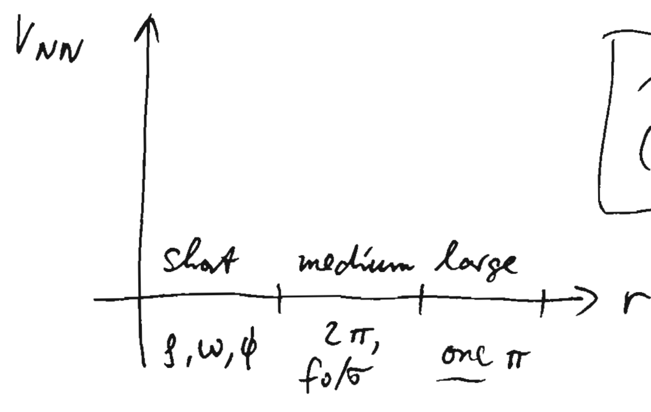
$$\Delta r \approx \lambda_c = \frac{\hbar c}{mc^2} = \frac{197.3\ \text{MeV}\cdot\text{fm}}{mc^2(\text{meson})}$$

<u>This implies the foll. picture:</u>

a) at <u>large distances</u>, V_{NN} is dominated by the lowest-mass meson, the <u>pion</u> with <u>$mc^2 \approx 140$ MeV</u>, resulting in a range of $\boxed{(\Delta r)_\pi = \frac{197.3\ \text{MeV}\cdot\text{fm}}{140\ \text{MeV}} \approx 1.4\ \text{fm} = \lambda_c^\pi}$

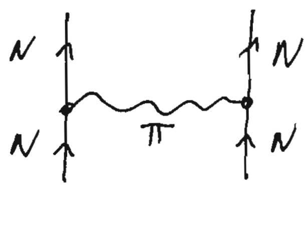
b) at <u>intermediate distances</u>, we can either have <u>two-pion</u> exchange or f_0/σ - meson exchange.

c) at <u>short distances</u>, we have <u>heavier meson</u> exchange (ρ, ω, ϕ)



meson contribution (qualitative picture)

The "OPEP" potential



From the Feynman diagram on the left one can determine the one-pion exchange potential (OPEP) in the static limit (<u>Details</u>: Bjorken & Drell, "Rel. Quantum Mechanics").

one finds

$$\boxed{\begin{aligned} V_{OPEP}(\vec{r},\vec{\sigma}_1,\vec{\sigma}_2,\vec{\tau}_1,\vec{\tau}_2) &= f_1(r)(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)+ \\ &\quad + f_2(r)(\vec{\tau}_1\cdot\vec{\tau}_2)\,S_{12} \\ \text{with } f_1(r) &= 0.088\cdot\left(\frac{m_\pi c^2}{3}\right)\frac{e^{-r/\lambda_c^\pi}}{(r/\lambda_c^\pi)} \\ \text{and } f_2(r) &= 0.088\cdot\left(\frac{m_\pi c^2}{3}\right)\frac{e^{-r/\lambda_c^\pi}}{(r/\lambda_c^\pi)}\left[1+\frac{3}{(r/\lambda_c^\pi)}+\frac{3}{(r/\lambda_c^\pi)^2}\right] \end{aligned}}$$

where the pion rest energy is $(m_\pi c^2) \approx 140\,\mathrm{MeV}$ and $\lambda_c^\pi = 1.4\,\mathrm{fm}$.

The term

$$\frac{e^{-r/\lambda_c^\pi}}{(r/\lambda_c^\pi)}$$

is the <u>famous "Yukawa potential"</u> predicted by Yukawa in 1935! We see that the first term in <u>V_{OPEP}</u> contributes to the <u>"spin-isospin component"</u> $O_4 = (\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)$ term ($\hat{=}\sigma\tau$) in the Argonne v_{18} potential, see <u>slide 2</u>.

We introduce now <u>further terms in v_{18}:</u>

$O_5 = S_{12}$ ("tensor" part, $\hat{=}\; t$)

$O_6 = S_{12}(\vec{\tau}_1\cdot\vec{\tau}_2)$ ("tensor-isospin" part $\hat{=}\; t\tau$)

$O_7 = \vec{L}\cdot\vec{S}$ ("spin-orbit" part $\hat{=}\; ls$)

$O_8 = \vec{L}\cdot\vec{S}(\vec{\tau}_1\cdot\vec{\tau}_2)$ ("spin-orbit isospin" part $\hat{=}\; ls\tau$)

⋮ + other terms

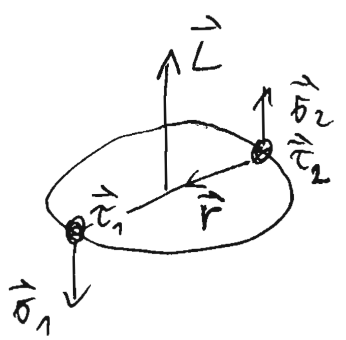
Apparently, V_{OPEP} contributes also a <u>tensor-isospin</u> component to v_{18}. The radial part of the O_5 and O_6 terms (labeled t and $t\tau$) are shown in $\boxed{\text{slide 3}}$. It also displays the OPEP contribution to the "$t\tau$" term O_6.

> Discuss slide 4: radial part for spin-orbit term O_7 ($\vec{L}\cdot\vec{S}$) and O_8 ($\vec{L}\cdot\vec{S}\,(\vec{\tau}_1\cdot\vec{\tau}_2)$) in Argonne v_{18} potential.

Can we understand the spin-orbit term on the basis of meson exchange theory? Yes! The foll. table gives the mesons and the type of interaction potential V_{NN} that they generate [Greiner & Maruhn textbook, p. 213]:

meson (J^π)	name	V_{NN} type
scalar (0^+)	"σ-meson" f_0/σ	1, $\vec{L}\cdot\vec{S}$
pseudoscalar (0^-)	π, η, η'	S_{12}
vector (1^-)	ρ, ω, ϕ	1, $\vec{\sigma}_1\cdot\vec{\sigma}_2$, S_{12}, $\vec{L}\cdot\vec{S}$

N-N quantum states and WF's



see also slide 8

We define the foll. quantities for the N-N pair:

$\vec{r} = \vec{r}_1 - \vec{r}_2$ = rel. distance

$\vec{p} = \vec{p}_1 - \vec{p}_2$ = rel. momentum

$\vec{L} = \vec{r} \times \vec{p}$ = rel. orbital ang. mom.

$\vec{S} = \frac{\hbar}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ = spin of N-N pair

$\vec{J} = \vec{L} + \vec{S}$ = total ang. mom. of N-N pair

$\vec{T} = \frac{1}{2}(\vec{\tau}_1 + \vec{\tau}_2)$ = isospin of N-N pair.