Nuclear many-particle Hamiltonian
(in coordinate representation)

$$H = \sum_{i=1}^{A} \frac{-\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} \sum_{i,j=1}^{Z} v_{ij}^{(2)Coul} + \frac{1}{2} \sum_{i,j=1}^{A} v_{ij}^{(2)nucl} + \frac{1}{6} \sum_{i,j,k=1}^{A} v_{ijk}^{(3)nucl}$$

Formal structure $H = T^{(1)} + V^{(2)} + V^{(3)}$

- kinetic energy of all nucleons (1-body operator)
- 2-body Coulomb interactions between protons
- 2-body and 3-body strong nuclear interactions

many-particle Schrödinger equation (in coordinate representation)

$(H - E_n)\Psi_n(x_1, x_2, ..., x_A) = 0$ with $x = (\vec{r}, s_z, t_z)$

Nuclear many-particle Hamiltonian
(in occupation number representation)

Creation operators for nucleon states: $\hat{c}_i^\dagger$

$$\hat{H} = \sum_{i,j=1}^{\infty} \langle i|t|j\rangle \hat{c}_i^\dagger \hat{c}_j + \frac{1}{2}\sum_{i,j,k,l=1}^{\infty} \langle ij|V^{(2)}|kl\rangle \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_l \hat{c}_k$$
$$+\frac{1}{6}\sum_{i,j,k,l,m,n=1}^{\infty} \langle ijk|V^{(3)}|lmn\rangle \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k^\dagger \hat{c}_n \hat{c}_m \hat{c}_l$$

Nuclear many-particle state vectors (occupation number rep.)

Basic building blocks: set of Slater determinants for A nucleons (anti-symmetrized products of single-particle wave functions)

These correspond to the state vectors in occupation number rep.

$$\Phi_{n_1,\ldots,n_\infty}(1,\ldots,A) = (\hat{c}_1^\dagger)^{n_1}\ (\hat{c}_2^\dagger)^{n_2}\ \ldots\ (\hat{c}_\infty^\dagger)^{n_\infty}|0>$$

$$with\ n_k = 0\ or\ 1 \quad and \quad \sum_{k=1}^{\infty} n_k = A$$

Theorem: the most general state vector of a many-body system can be written as an infinite superposition of Slater determinants

$$\Psi(1,\ldots,A;t) = \sum_{n_1,\ldots,n_\infty=0}^{1} f(n_1,\ldots,n_\infty;t)\ \Phi_{n_1,\ldots,n_\infty}(1,\ldots,A)$$

Numerical solution of nuclear many-particle problem

Need to truncate infinite set of Slater determinants.

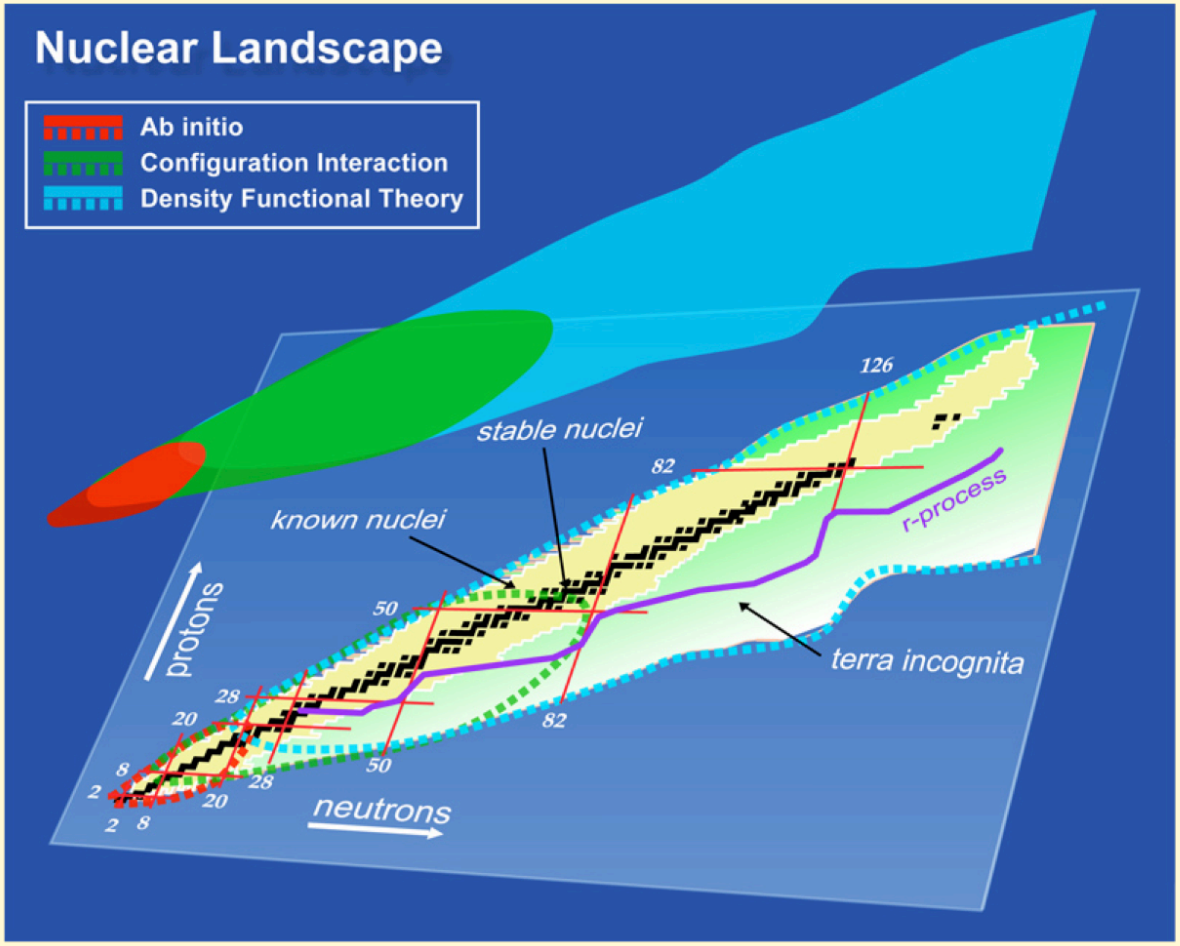
"Ab-initio" methods use of the order of 10^5 to 10^6 Slater determinants.

Need largest available supercomputers (Livermore, ORNL, …)

Use quasi-particle transformation to "particle" states (above Fermi level) and "hole" states (below Fermi level); state vector for ground state becomes "quasi-particle" vacuum.

Then use Wick's theorem to simplify matrix elements of creation and annihilation operators.

Microscopic theories of nuclear structure



“Ab-initio” calculations for light nuclei

- Green's function Monte Carlo method
- “No-core” shell model (no inert core, many-body theory for all nucleons)
- Coupled-cluster theory (based on Goldstone’s “linked cluster expansion”)

Green's function Monte Carlo calculations for light nuclei using Argonne v18 + 3-body interaction

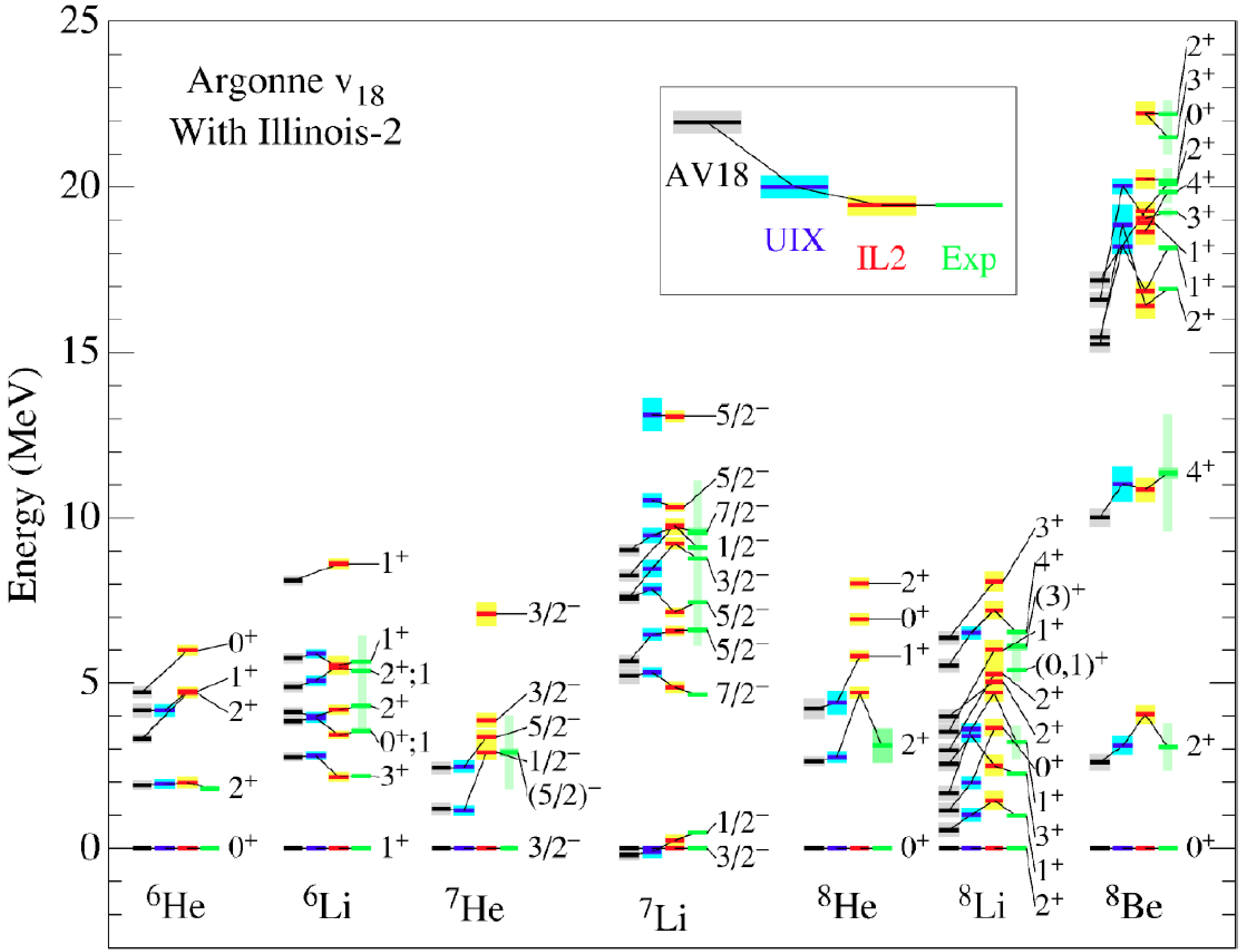
Ref: Pieper, Wiringa & Carlson, Phys. Rev. C 70, 054325 (2004)

first step: variational Monte Carlo (VMC) calculation to find an optimal many-body trial wave function for a given state. The trial WF contains variational parameters that are adjusted to minimize the energy expectation value, which is evaluated by METROPOLIS Monte Carlo integration.

second step: the trial wave function (antisymmetric by explicit construction) is used as input to a Green's function Monte Carlo (GFMC) algorithm which projects out the lowest energy eigenstate by a propagation in imaginary time.

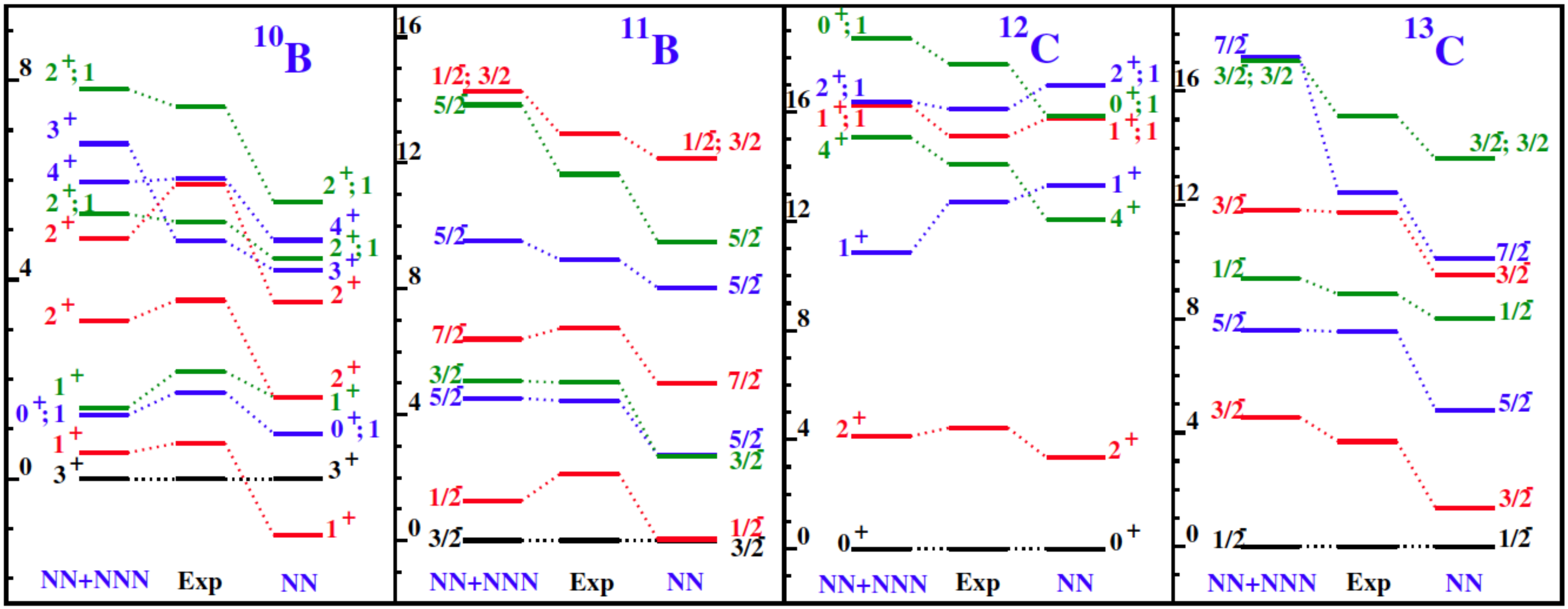
See figure next page!

NOTE: inclusion of 3-body N-N interaction (labeled IL2, red-yellow) improves agreement between theory and exp. data!


Argonne v18
With Illinois-2
AV18
UIX
IL2
Exp
Energy (MeV)
25
20
15
10
5
0
6He
6Li
7He
7Li
8He
8Li
8Be

No-core shell model

Ref: Navratil, Gueorguiev, Vary, Ormand and Nogga,
Phys. Rev. Lett. 99, 042501 (2007)



Coupled-cluster theory

Ref: Hagen, Papenbrock, Dean, and Hjorth-Jensen
Phys. Rev. Lett. 101, 092502 (2008)

TABLE I. CCSD results for various nuclei from a chiral nucleon-nucleon potential at order NNNLO. E/A and V/A: ground-state and potential energy per nucleon, respectively. $\Delta E/A$: difference to the experimental ground-state energy (theoretical mass table evaluations for ^{48}Ni). R and R_{exp} are the computed and measured charge radius.

Nucleus	E/A [MeV]	V/A [MeV]	$\Delta E/A$ [MeV]	R [fm]	R_{exp} [fm]
^{4}He	−5.99	−22.75	1.08	1.86	1.64
^{16}O	−6.72	−30.69	1.25	2.71	2.74
^{40}Ca	−7.72	−36.40	0.84	3.24	3.48
^{48}Ca	−7.40	−37.97	1.27	3.22	3.47
^{48}Ni	−6.02	−36.04	1.21	3.50	