Microscopic theory of collective vibrations: linear response theory (RPA, QRPA)

Ref: Ring & Schuck, chapter 8

Motivation

Mean field theories (HF, HFB) describe independent particle motion, explain basic properties of ground states very well.

Excited states, however, show new phenomena: in addition to single-particle excitations built on the HF / HFB ground state, one finds collective vibrations which represent a "coherent" motion of many nucleons:

Low-energy vibrations (E* = few MeV) ⇒ surface oscillations (L=2,3, …)

High-energy vibrations (E* = 10-30 MeV) ⇒ density oscillations = "giant resonances" (L=0,1,2,3, …)

Random Phase Approximation (RPA)

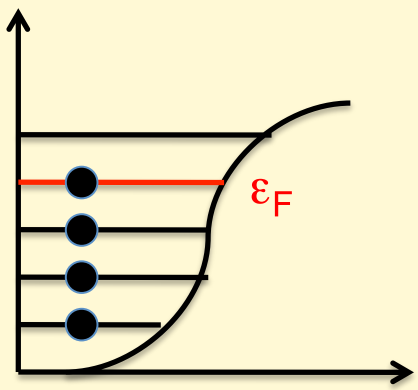
RPA theory was introduced by Bohm and Pines (1953) to describe plasma oscillations.

In nuclear physics it is used to describe collective vibrations.

These collective vibrations are caused by the long-range part of the residual interaction which leads to particle-hole (p-h) correlations.

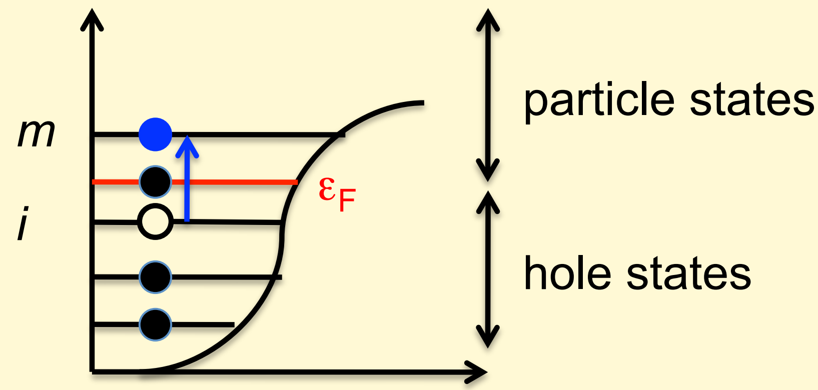
If pairing is included also, the theory is called the Quasi-particle Random Phase Approximation (QRPA).

particle-hole excitations



uncorrelated
HF ground state

$$|HF>$$



1p-1h excitation

$$\hat{c}_m^{+}\hat{c}_i\,|HF>$$

Structure of RPA ground state and excited states

Collective vibration = coherent superposition of large number of p-h excitations

$$|\nu_{RPA}> = \sum_{m,i} A^{\nu}_{mi} (\hat{c}_m{}^{+} \hat{c}_i) \,|HF>$$

where most amplitudes *A* in the equation above contribute with the same sign.

In RPA theory, one takes into account p-h correlations not only in excited states, but also in the ground state. It can be shown that the RPA ground state contains virtual 2p-2h correlations (Ring & Schuck, p. 310):

$$|0_{RPA}> = |HF> + \sum_{m,i,n,j} B^{0}_{mi,nj} (\hat{c}_m{}^{+} \hat{c}_i)(\hat{c}_n{}^{+} \hat{c}_j) \,|HF>$$

Derivation of RPA equations from Time-dependent Hartree-Fock equations: Linear Response Theory

Ref: Ring & Schuck, p. 315-316

We add a time-dependent external field *F(t)* to the nuclear Hamiltonian

$$\hat{H}(t) = \hat{H}_0 + \hat{F}(t)$$

F(t) is supposed to be a one-body operator (e.g. electric multipole operator) and varies harmonically with time at a given frequency ω

$$\hat{F}(t) = \sum_{k,l} \left[f_{kl} e^{-i\omega t} + f_{kl}{}^{+} e^{+i\omega t} \right] \hat{c}_k{}^{+} \hat{c}_l$$

Consider the time-dependent state vector of the system $|\Phi(t)>$

with associated one-body density

$$\rho_{kl}(t) = <\Phi(t)| \hat{c}_l{}^{+} \hat{c}_k |\Phi(t)>$$

Derivation of RPA equations from Time-dependent Hartree-Fock equations: Linear Response Theory

Approximation: assume state vector is single Slater determinant

$$|\Phi(t)\rangle = \det[\varphi_i(t)]$$

This yields the Time-Dependent Hartree-Fock (TDHF) equations;
for derivation see e.g. Ring & Schuck, p. 485-488

$$\left[h(\rho) + f(t), \rho\right] = i\hbar \partial\rho(t) / \partial t$$

In the limit that the time-dep. external field is weak, one obtains a small-amplitude density vibration

$$\rho(t) = \rho^{(0)} + \delta\rho(t)$$

$$\delta\rho(t) = \rho^{(1)} e^{-i\omega t} + \rho^{(1)^{+}} e^{+i\omega t}$$

Derivation of RPA equations from Time-dependent Hartree-Fock equations: Linear Response Theory

In the small-amplitude vibrational limit, the TDHF equations yield the "linear response equations"

$$\rho^{(1)} = R(\omega) f$$

$$\rho_{kl}^{(1)} = \sum_{p,q} R_{kl,pq}(\omega) f_{pq}$$

Note: Response function R contains info about residual interaction

$$\tilde{v}_{kj,li} = \frac{\partial h_{kl}}{\partial \rho_{ij}}$$

density vibration ("response") | Response function | ext. field

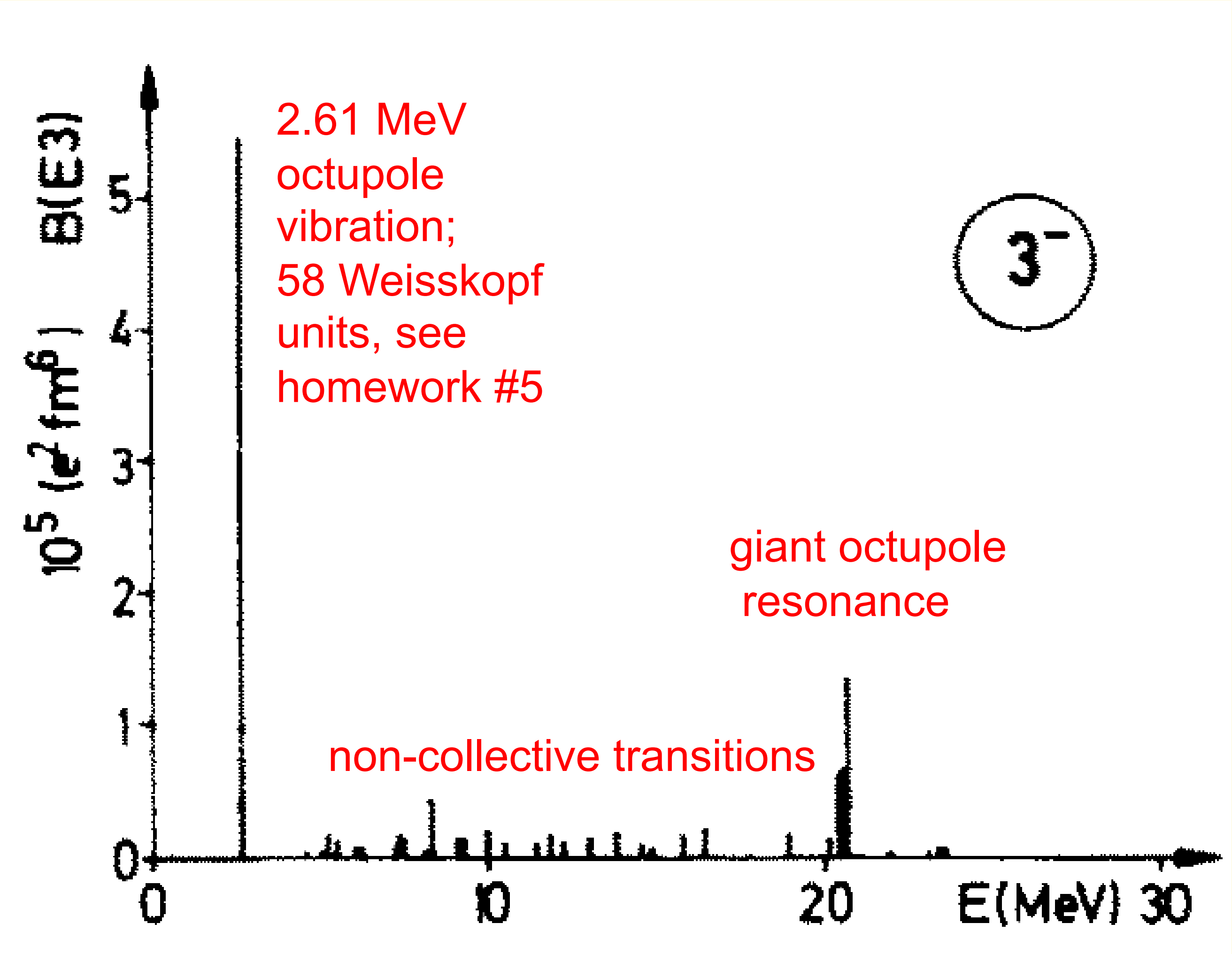
The RPA equations correspond to the homogeneous part of the "linear response equations" (i.e. *f=0*); eigenvalue problem for $\omega = \omega_n$

$$R^{-1}(\omega)\rho^{(1)} = 0 \qquad E_n = \hbar\omega_n$$

The RPA equations determine the energies of the RPA ground state and excited states and their transition strengths.

Early RPA calculations for ^{208}Pb

P. Ring and J. Speth, Nucl. Phys. A 235 (1974) 315



Approximations:

Woods-Saxon shell model

use 2 main shells above and below Fermi surface

density-dependent delta interaction

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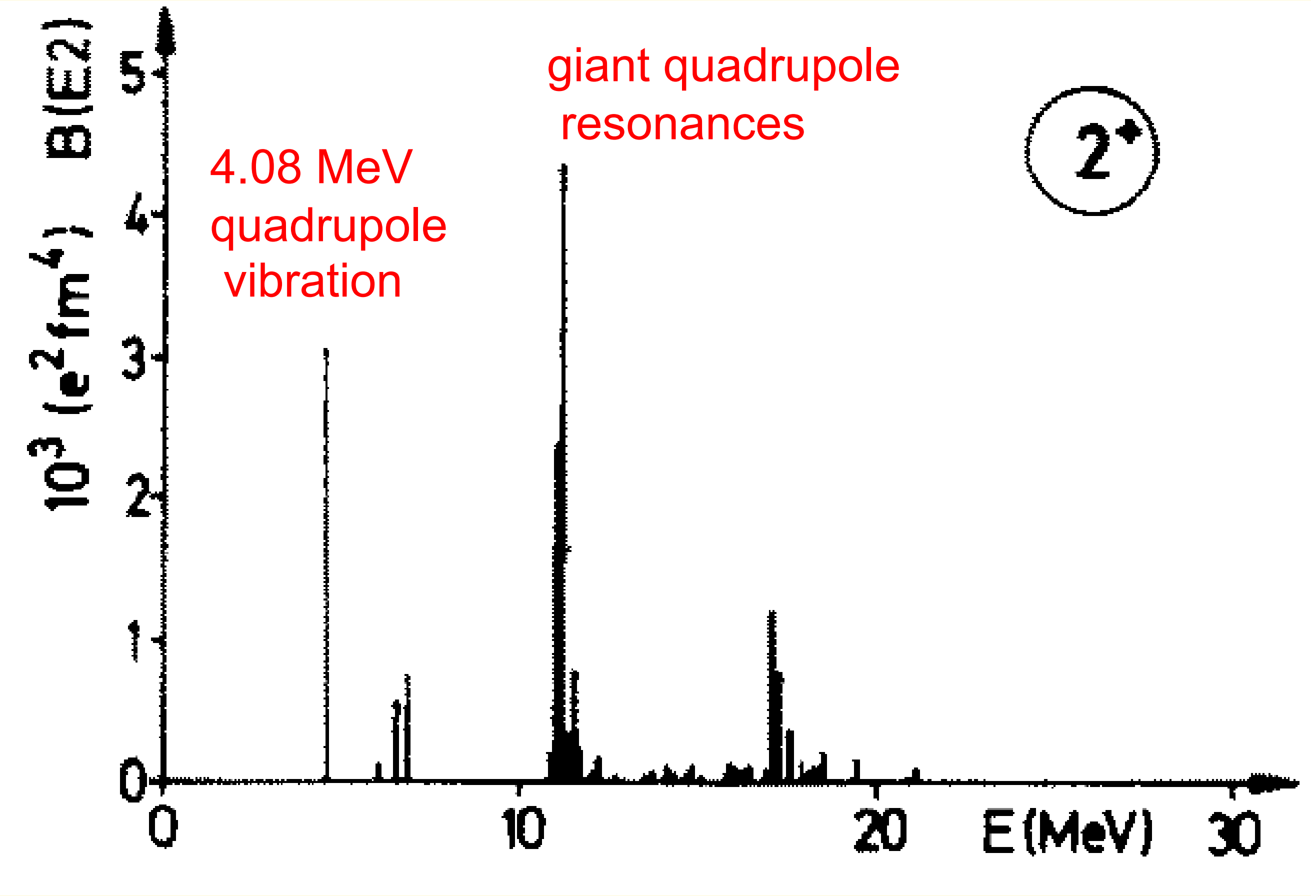

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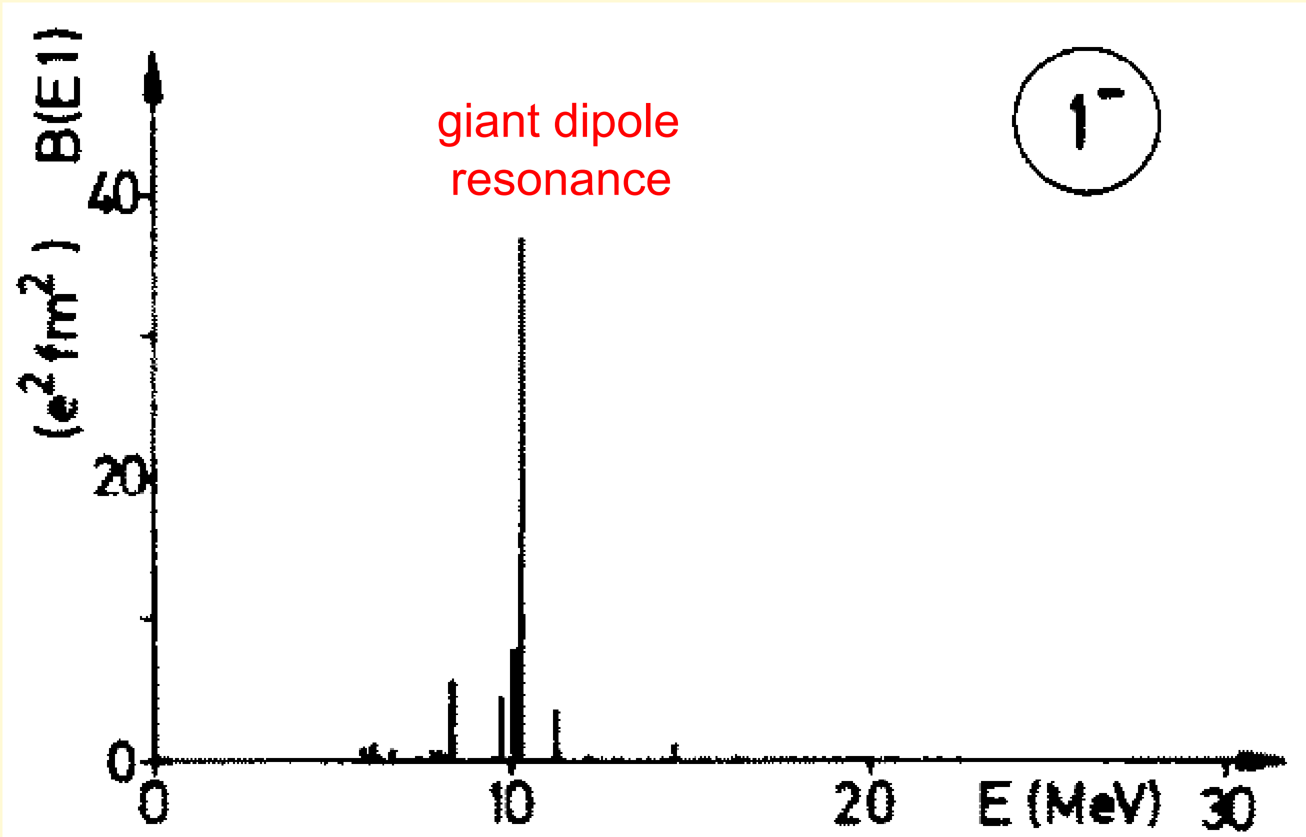

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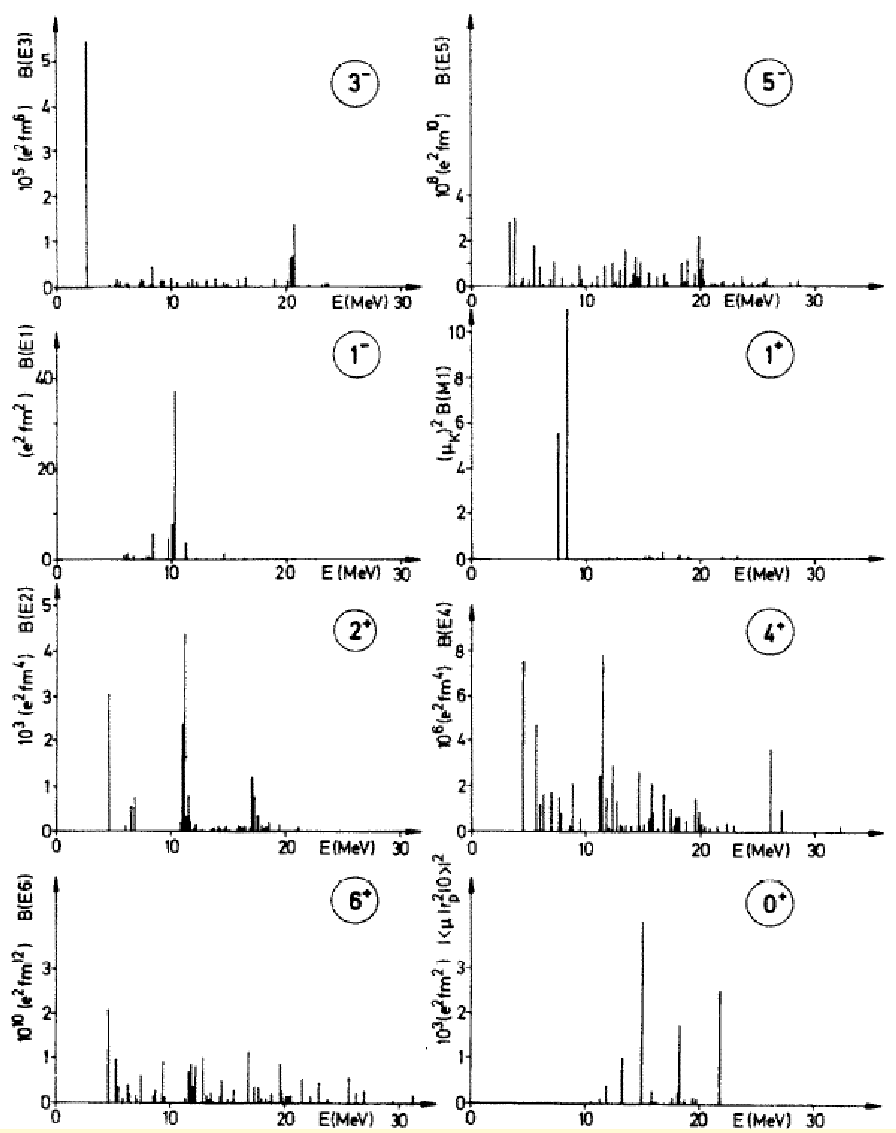


All multipolarities (L = 0 – 6)

Linear response based on 3-D TDHF (apply opposite boosts to protons and neutrons)

Maruhn et al., Phys. Rev. C71, 064328 (2005)

DIPOLE GIANT RESONANCES IN DEFORMED HEAVY NUCLEI

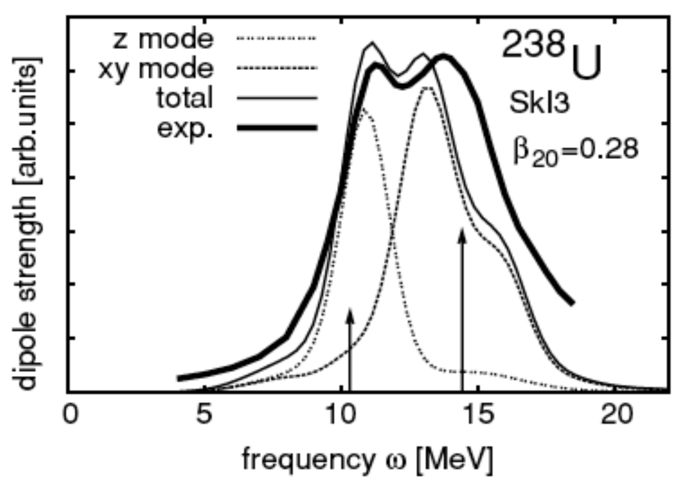
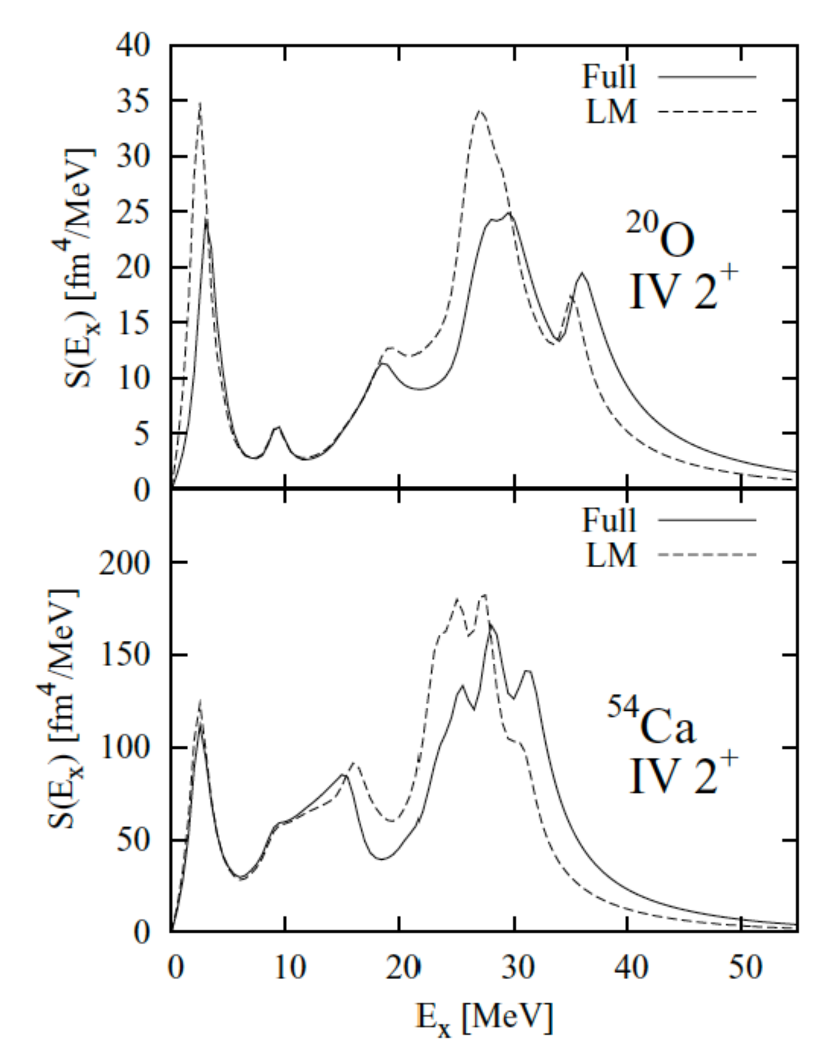


FIG. 7. Dipole strength in the prolate ground state of ^{238}U computed with SkI3. The total strength is shown as well as the strengths for the separate modes. The experimental result from [3] is shown for comparison. The estimated peak positions (see Sec. III A) are indicated by arrows.

Continuum quasi-particle linear response theory for neutron-rich isotopes

Mizuyama, Matsuo & Serizawa, Phys. Rev. C79, 024313 (2009)



Strength function of isovector (T=1) quadrupole (L=2) response.

(LM = Landau-Migdal approx.)

Theory yields both low-energy and high-energy collective vibrations

Skyrme (SkM*) interaction for HFB mean field, and density-dependent delta interaction for pairing