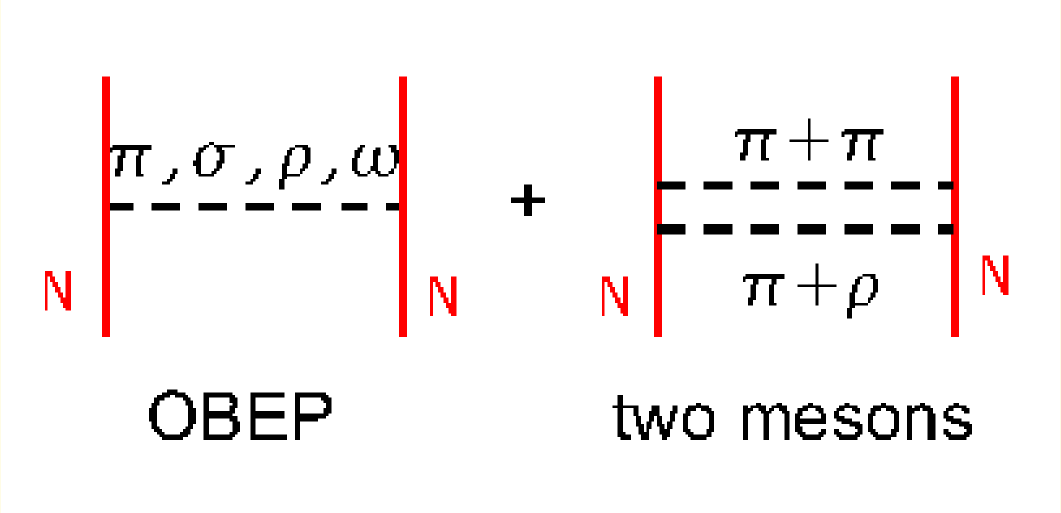
Derive N-N interaction potentials from meson exchange theories



Most important: 1-pion exchange potential (OPEP) contributes to spin-isospin (σ τ) and tensor-isospin (t τ) components of Argonne v-18 potential

Experimental p-p and n-p differential cross sections

Ref: W.N. Hess, Rev. Mod. Phys. 30, 368 (1958)

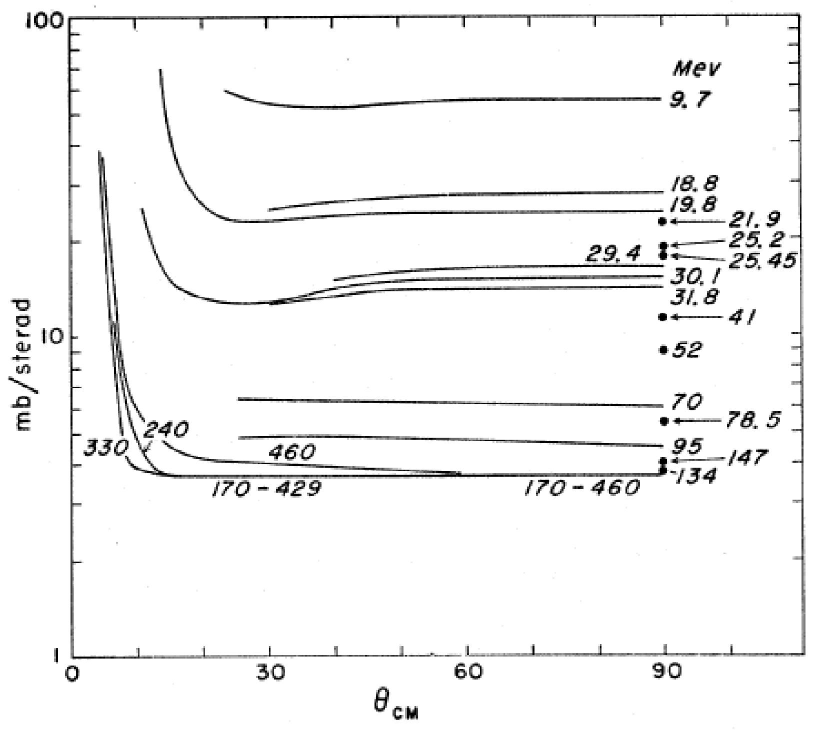


FIG. 6. Experimental values of the differential proton-proton cross section at various energies up to 500 Mev.

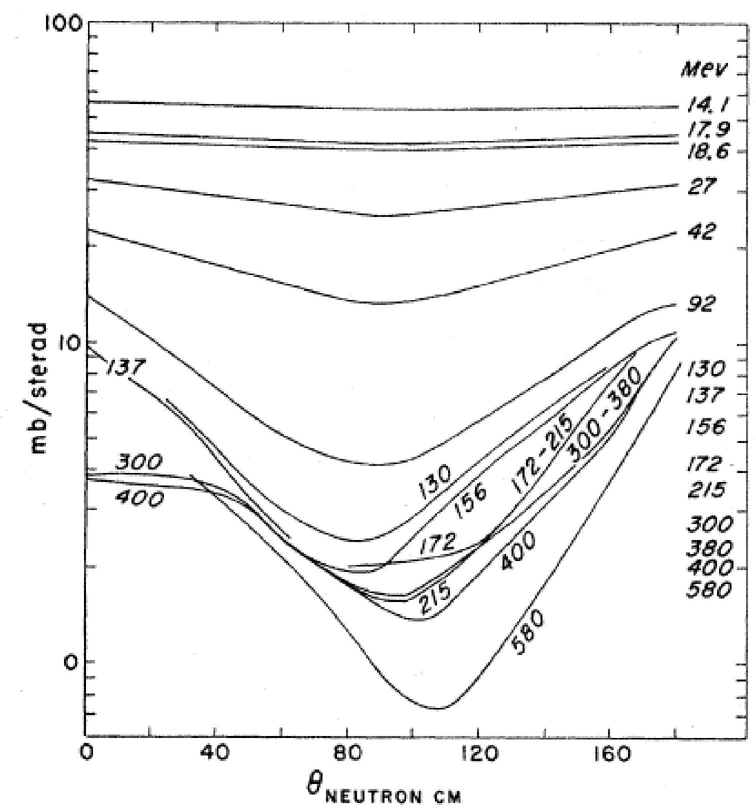
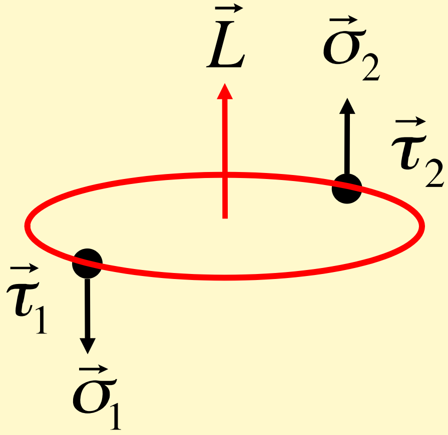


FIG. 5. Experimental values of the differential neutron-proton cross section at various energies.

N-N quantum states



$\vec{L} = \vec{r} \times \vec{p}$ orbital ang. momentum

$\vec{S} = \frac{\hbar}{2}\left(\vec{\sigma}_1 + \vec{\sigma}_2\right)$ spin of N-N pair

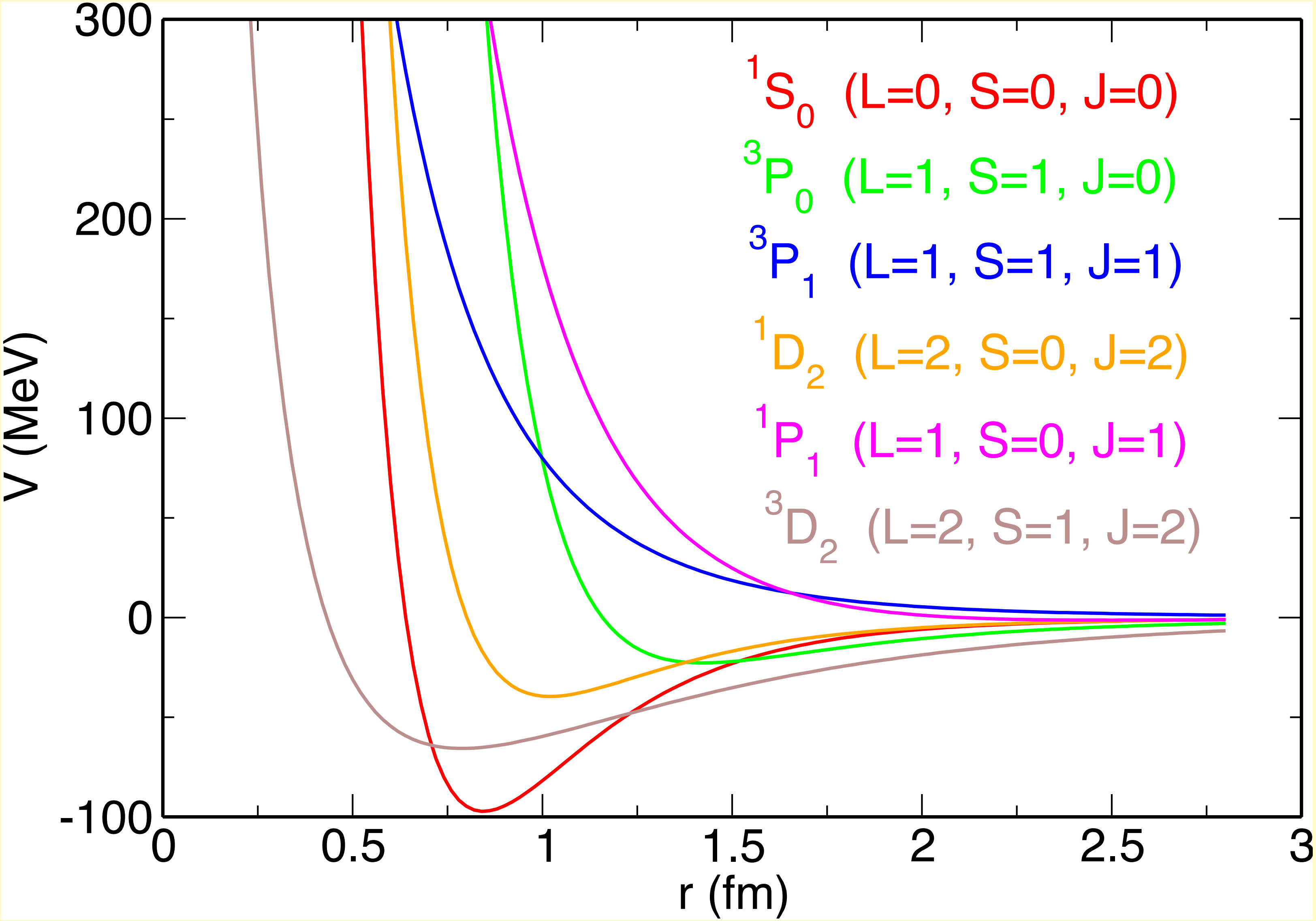
$\vec{J} = \vec{L} + \vec{S}$ total ang. momentum

$\vec{T} = \frac{1}{2}\left(\vec{\tau}_1 + \vec{\tau}_2\right)$ isospin of N-N pair

Spectroscopic notation: ${}^{(2S+1)}L_J$ use S,P,D,... for L=0,1,2,...

N-N state vector: $\left|\Psi(1,2)\right\rangle = \left|LS;J\,M_J\right\rangle \otimes \left|T,T_z\right\rangle$

Free N-N interaction: Reid soft-core potential (1968) for some of the reaction channels



Need for "effective" N-N interaction (in nuclear medium)

Computational reason:
N-N potentials exhibit, for some reaction channels, a strongly repulsive core (≈ 4000 MeV) at r ≈ 0.5 fm. Potential becomes very large, wave function becomes very small. This is numerically unstable.

Many-body physics reasons:
For free N-N scattering, almost all quantum states are unoccupied; in a heavy nucleus, however, many quantum states are occupied and thus "Pauli-blocked" (scattering into these states is forbidden).

For free N-N scattering, the energy of the N-N pair is conserved, by for N-N scattering in nuclear medium the energy of N-N pair is not conserved (energy transfer to other nucleons).

Derive effective interaction (Brückner G-matrix) from Bethe-Goldstone equation

Ref: Ring & Schuck, chapter 4.3.1

G-matrix = effective interaction | free N-N interaction

$$< ab|G^E|cd> = < ab|\bar{V}|cd> +$$

$$\frac{1}{2}\sum_{m,n>\varepsilon_F} < ab|\bar{V}|mn> \frac{1}{E-\varepsilon_m-\varepsilon_n+i\eta} < mn|G^E|cd>$$

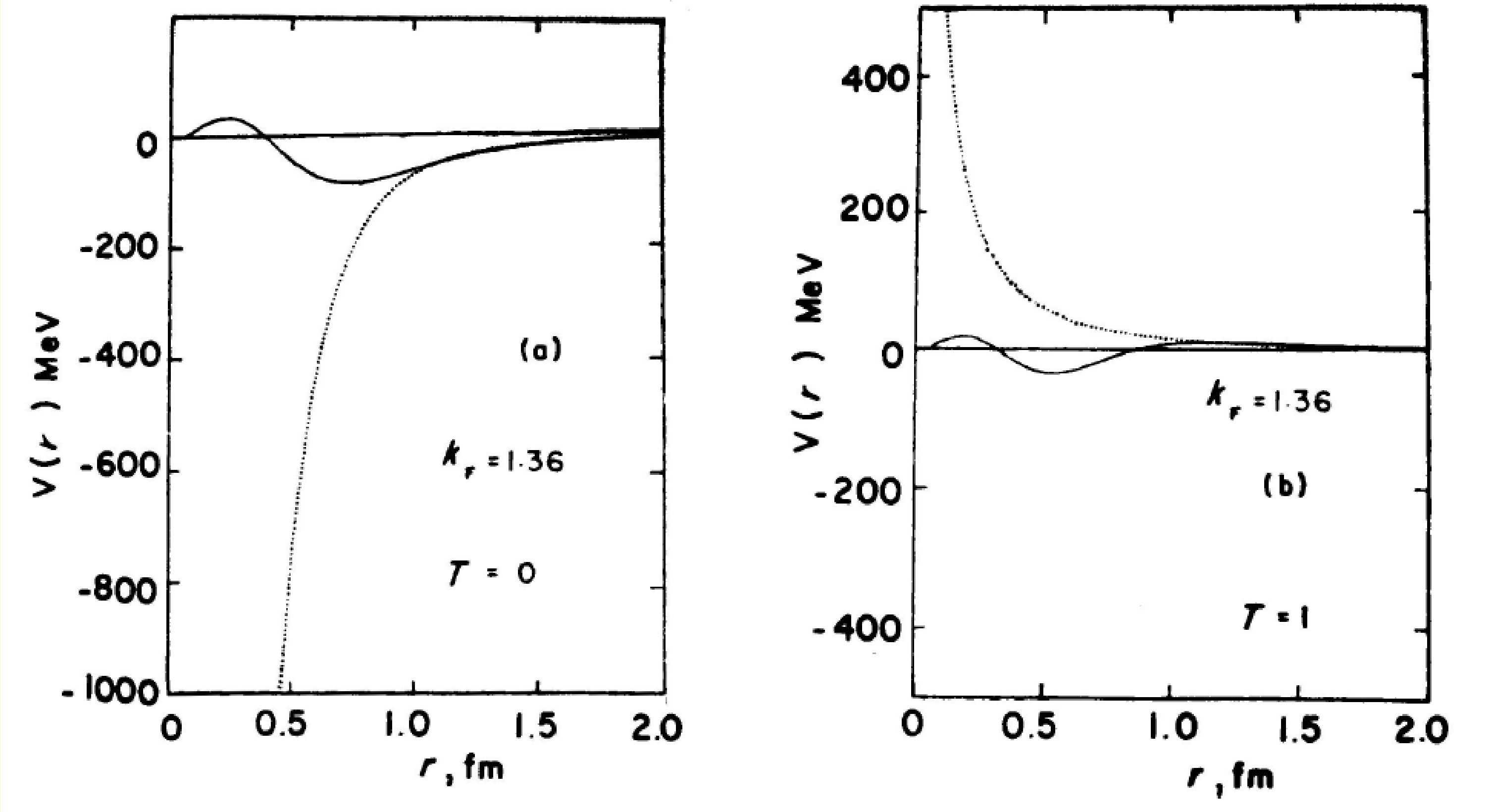
Pauli blocking | free N-N interaction | single-particle energies | G-matrix = effective interaction

This equation must be solved iteratively; not too hard for infinite nuclear medium (“nuclear matter”) but fairly difficult for finite nuclei !

Example: tensor components of Reid soft-core N-N interaction

Ref: Sprung and Banerjee, Nucl. Phys. A168, 273 (1971)

solid line: "effective" interaction in nuclear matter, from Bethe-Goldstone eq.
dotted line: free N-N interaction



Comments on effective N-N interaction

From the numerical results depicted in the last slide we conclude:

- At distances r > 1.0 fm , the free N-N interaction and the effective interaction are identical !

- At distances r < 1.0 fm, however, the free N-N interaction may become extremely large (almost singular) while the corresponding effective N-N interaction is finite everywhere ! This is primarily due to “Pauli blocking”.

- Therefore, the effective interaction is a better starting point for numerical calculations, in particular for mean-field theories (HF, HFB) of heavy nuclei.