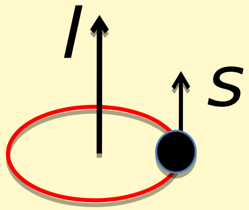
# Cooper pair formation in atomic nuclei

Experiments show that all nuclei with even number of protons and even number of neutrons have a ground state with total angular momentum J=0.

Interpretation: this suggests a pairwise coupling of nucleon angular momenta to j(pair)=0.




$$j = l + s$$

$$j(pair) = j_1 + j_2 = 0$$

Nuclear mean field theory (Hartree Fock) does not describe this pairing phenomenon. We need to go beyond mean field physics and consider the residual interaction between nucleons.

# Nuclear mean field and residual interaction

original Hamiltonian $H = T^{(1)} + V^{(2)} + V^{(3)}$

add and subtract a 1-body "mean field potential" $V^{(1)}$
(derive from Hartree-Fock theory)

$$H = T^{(1)} + V^{(1)} + V^{(2)} + V^{(3)} - V^{(1)}$$

regroup and separate terms

$H = H_{mf} + H_{res}$ complete many-body Hamiltonian

$H_{mf} = T^{(1)} + V^{(1)}$ mean field Hamiltonian, 1-body central field

$H_{res} = V^{(2)} + V^{(3)} - V^{(1)}$ residual interaction, "small"

# Nuclear mean field and residual interaction

textbook by Ring & Schuck, chapters 4 and 6

residual interaction $H_{res} = V^{(2)} + V^{(3)} - V^{(1)}$

long-range part causes particle-hole (p-h) correlations:

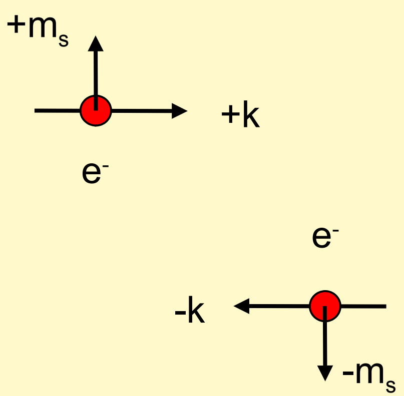
contributes to nuclear deformation (quadrupole-quadrupole part of residual interaction)

short-range part causes particle-particle (p-p) correlations:

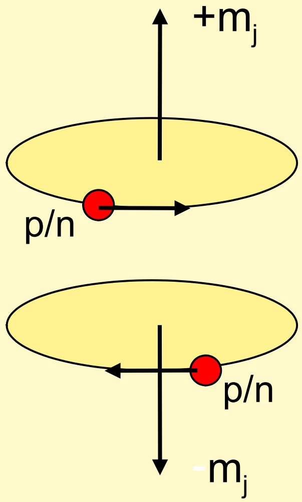
causes Cooper pair formation, pairing vibrations

# Cooper pair formation

Condensed matter physics:
electron-phonon interaction




Nuclear physics:
short-range residual interaction



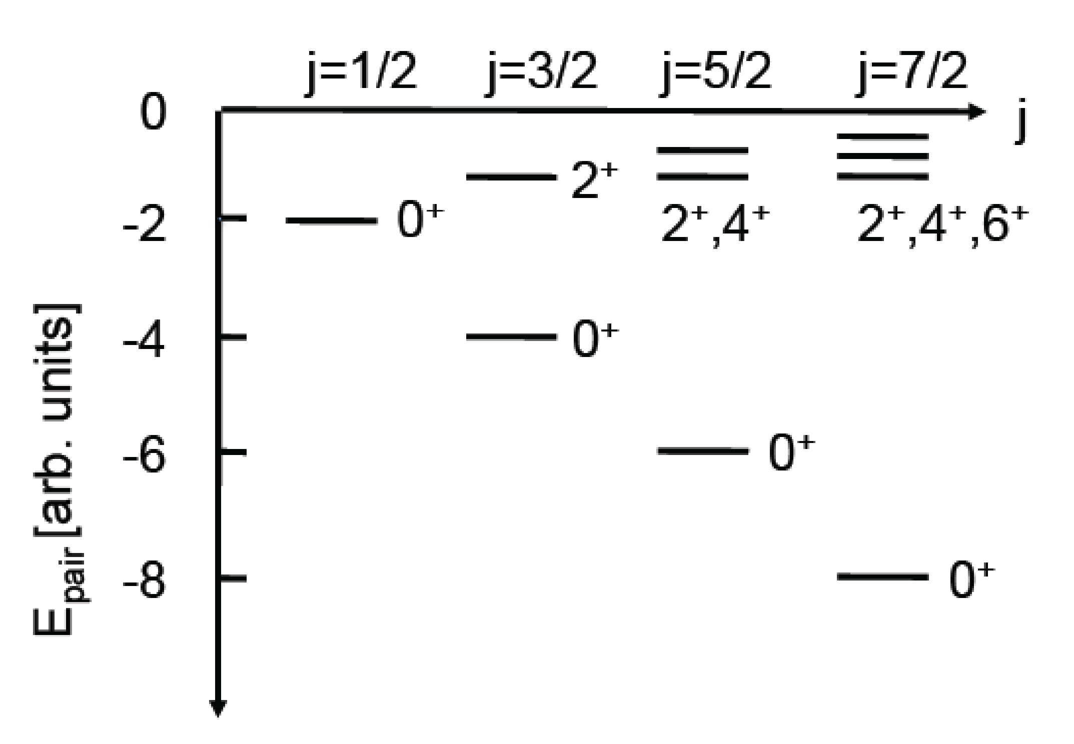
# Short-range residual interaction: pairing energy of 2 nucleons in the same j-shell

Ref: Fetter & Walecka, p. 515-519

Approximate short-range interaction by attractive delta function

$$V(1,2) = -G\ \delta(\vec{r}_1 - \vec{r}_2)$$

Assume 2 identical particles in the same j-shell; binding energy of pair is largest for J (pair) = 0.




This simple model explains qualitatively the experimental fact that all even-even nuclei have ground state angular momentum J = 0.

# BCS pairing model for nuclei

Ref: Fetter and Walecka, p. 337

Notation: $k = |j, +m_j>$ $\quad -k = |j, -m_j>$

time-reversed state

BCS model assumes the following structure of a paired ground state (based on Cooper's model for a single pair)

$$|BCS> = \prod_{k>0}^{\infty} (u_k \; + \; v_k \; \hat{c}_k^\dagger \; \hat{c}_{-k}^\dagger) \; |0>$$

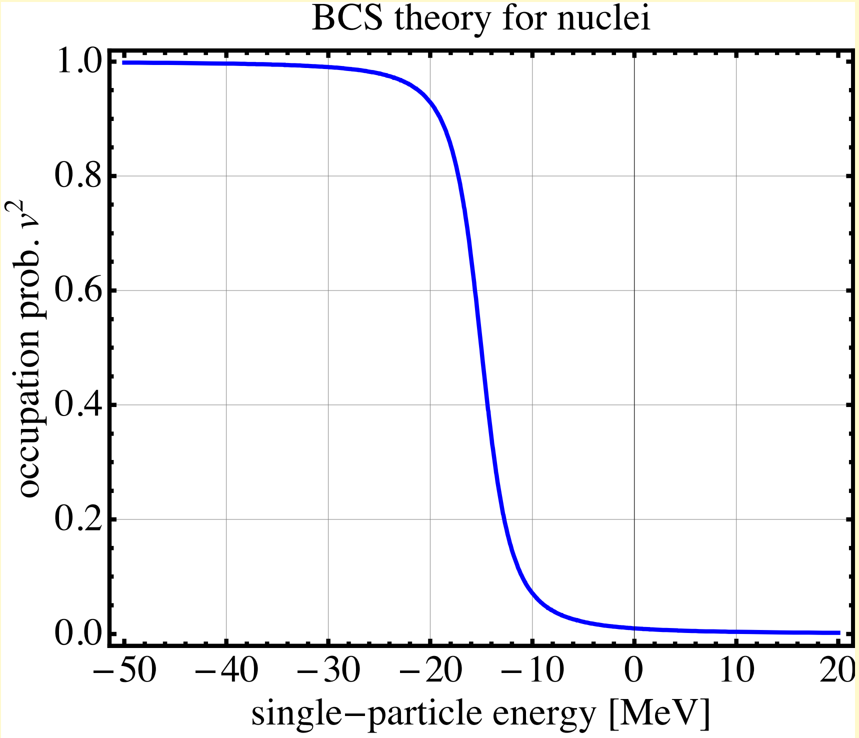
$v_k^2 =$ probability that single-particle levels (k, -k) are occupied

$u_k^2 =$ probability that single-particle levels (k, -k) are empty

# BCS pairing for nuclei: occupation probability for states (k,-k)

Ref: Fetter & Walecka, p. 334

$$v_k^2 = \frac{1}{2}\left[1 - \frac{(\epsilon_k - \lambda)}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right]$$




Pairing forces lead to fractional occupation numbers.
The Fermi surface is no longer sharp, but becomes "soft".

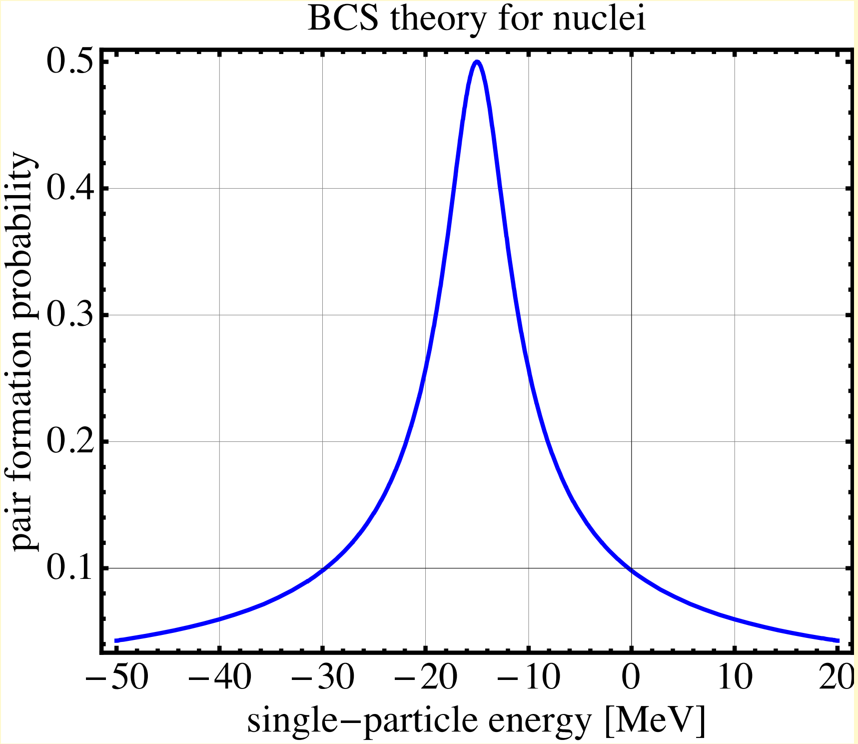
$\varepsilon_k$= single-particle energy

λ = -15 MeV (generalized Fermi energy)

Δ = 3 MeV (pairing gap, determines width of distribution)

# BCS pairing for nuclei: spectral distribution of pairing density

Ref: Fetter and Walecka, p. 337




$$P_k = < BCS|\ \hat{c}_k^\dagger\ \hat{c}_{-k}^\dagger\ |BCS >$$

$$= v_k\ u_k = v_k \sqrt{1 - v_k^2}$$

λ = -15 MeV (generalized Fermi energy)

Note: pair formation is concentrated in the vicinity of the Fermi level

# BCS pairing Hamiltonian and particle density

Ref: Ring & Schuck, p. 232

$$\hat{H} = \sum_k \epsilon_k \; \hat{c}_k^\dagger \; \hat{c}_k - G \sum_{k,k'>0} \hat{c}_k^\dagger \; \hat{c}_{-k}^\dagger \; \hat{c}_{-k'} \; \hat{c}_{k'}$$

Single-particle energies may be obtained from HF theory
Constant pairing strength adjusted to exp. data

BCS theory shows that HF particle density

$$\rho^{HF}(\vec{r}, \sigma_z) = \sum_{k=1}^{A} |\phi_k(\vec{r}, \sigma_z)|^2$$

should be replaced by $\rho^{HF+BCS}(\vec{r}, \sigma_z) = \sum_{k=1}^{\infty} v_k^2 \; |\phi_k(\vec{r}, \sigma_z)|^2$