Section 1.3 Notes: nuclear units and constants of nature, with examples

Nuclear units

- length unit: Fermi = femtometer = fm = 10^{-15}m.
- energy unit: MeV or GeV.
- time unit: either second [s] or [fm/c].

Constants of nature relevant to nuclear physics (July 2000 Particle Physics Booklet)

- speed of light in vacuum, $c = 2.99792458 \times 10^{23}$ [fm/s]
- Planck's constant $/2\pi = \hbar = 6.58211889 \times 10^{-22}$ [MeV s]
- $\hbar c = 197.3269602$ [MeV fm]
- fine structure constant (dimensionless), $\alpha = e^2/(\hbar c) = 1/137.03599976$
- rest energy of proton, $m_p c^2 = 938.27200$ [MeV]
- rest energy of neutron, $m_n c^2 = 939.56533$ [MeV]

Examples

- Nuclear time unit [fm/c]

  From the constants given above we compute

  $$1\frac{fm}{c} \approx \frac{1[fm]}{3 \times 10^{23}[fm/s]} = 3 \times 10^{-24}[s] \ .$$

  This time unit corresponds to the time Δt it takes for a photon to travel a distance of 1 fm. It is indeed a useful time unit. For example, the Vanderbilt TDHF nuclear reaction code uses a time step of $0.4 fm/c$.

- Elementary charge e in nuclear units

  Let us look at the Coulomb potential energy (in Gaussian units) of two pointlike protons given by

  $$V_{Coul}[MeV] = \frac{e^2[?]}{r[fm]}$$

  From this, we conclude that the proper nuclear units for e^2 are [MeV fm]. We can compute e^2 using the constants of nature given above:

  $$e^2 = \alpha \ \hbar c = 1.4399643929[MeV \ \ fm]$$

  from which we conclude that the elementary charge e in nuclear units is given by

  $$e = 1.1999851636[\sqrt{MeV \ \ fm}]$$

- Kinetic energy operator constant in nuclear units

  In quantum mechanics, the kinetic energy operator for a particle of mass m is given by

$$-\frac{\hbar^2}{2m}\nabla^2$$

  Let us compute the constant quantity in this operator for a proton:

$$\frac{\hbar^2}{2m_p} = \frac{\hbar^2 c^2}{2m_p c^2} = \frac{(\hbar c)^2}{2\ m_p c^2} \approx \frac{197.3^2}{2 \times 938.3}\frac{[MeV^2\ fm^2]}{[MeV]} = 20.74[MeV\ fm^2]$$

- Compton wavelength of a neutron

  If one converts the Schrödinger equation (or Dirac equation) into dimensionless length and time units, then it turns out that the "natural" unit of length for a particle with mass m is the "reduced Compton wavelength" λ_C defined as

$$\lambda_C = \frac{\hbar}{mc} = \frac{\hbar c}{mc^2}\ .$$

  For a neutron we obtain

$$\lambda_C = \frac{\hbar c}{m_n c^2} = \frac{197.3[MeV\ fm]}{939.6[MeV]} = 0.21[fm]$$

  which is small compared to the diameter of a neutron (about 1.6 fm).