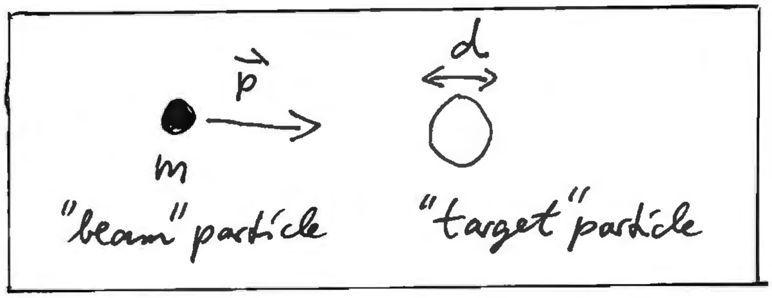
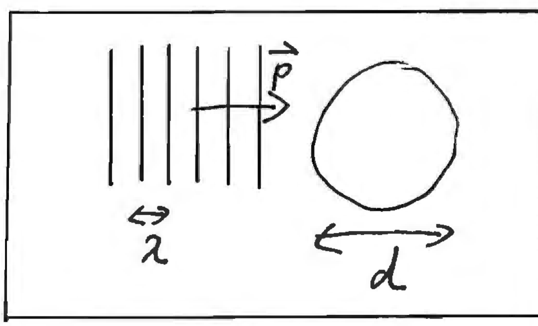
# 1.6 de Broglie wavelength of electron and heavy-ion beams

Most <u>information</u> about atomic nuclei and their constituents (nucleons / quarks) is obtained via <u>accelerators</u>. Consider a "beam" particle of rest mass $m$, momentum $\vec{p}$, and kinetic energy $T$ hitting a "target" particle of size $d$.




According to standard QM, the beam particles have wave properties, characterized by their de <u>Broglie wavelength</u> <u>$\lambda = h/p$.</u> From wave optics, we know that the beam particles can "resolve" an object of diameter $d$ provided that




$$\boxed{\lambda \ll d} \qquad \underline{(1)}$$

Typically, one specifies the kinetic energy $T$ of the beam particles. We use relativistic kinematics to compute $p$:

$$\boxed{\underbrace{E}_{\substack{\text{total}\\ \text{energy}}} = \underbrace{T}_{\substack{\text{kinetic}\\ \text{energy}}} + \underbrace{mc^2}_{\substack{\text{rest}\\ \text{energy}}}} \qquad \underline{(2a)}$$

From $E^2 = \vec{p}^{\,2}c^2 + (mc^2)^2$ we obtain $pc = \sqrt{E^2 - (mc^2)^2}$.

$$\Rightarrow \boxed{\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{2\pi\,(\hbar c)}{\sqrt{E^2 - (mc^2)^2}}} \qquad \underline{(2b)}$$

with $\boxed{\hbar c = 197.3 \text{ MeV}\cdot\text{fm}}$ $\qquad \underline{(2c)}$

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Application: compute $\lambda$ for nucleon bound in nucleus

nucleus

From the Fermi gas model (see section 2.2, p.18) one obtains a <u>max.</u> kinetic energy

$T_{max} = T_F \approx 37$ MeV.

$\Rightarrow \underline{E_{max}} = T_{max} + mc^2 = 37 + 939 = \underline{976 \text{ MeV}}$.

We obtain the <u>minimum</u> de Broglie wavelength of a nucleon bound in an atomic nucleus

$$\underline{\lambda_{min}} = \frac{2\pi(\hbar c)}{\sqrt{E_{max}^2 - (mc^2)^2}} = \frac{2\pi \times 197.3 \;\cancel{\text{MeV}}\cdot\text{fm}}{\sqrt{976^2 - 939^2}\;\cancel{\text{MeV}}} = \underline{4.6 \text{ fm}}$$

From scattering with polarized $e^-$, one obtains the charge distribution of p/n. For the proton, one finds a root-mean square radius:

$$r_{rms} = \sqrt{\langle r^2 \rangle} \simeq 0.8 \text{ fm} \Rightarrow d = 2r \simeq 1.6 \text{ fm}$$

$$\Rightarrow \boxed{\frac{\lambda_{min}}{d} = \frac{4.6 \text{ fm}}{1.6 \text{ fm}} \simeq 3}$$

We see that $\lambda_{min} > d$, so the nucleons inside a nucleus "see" each other as <u>pointlike particles</u>!