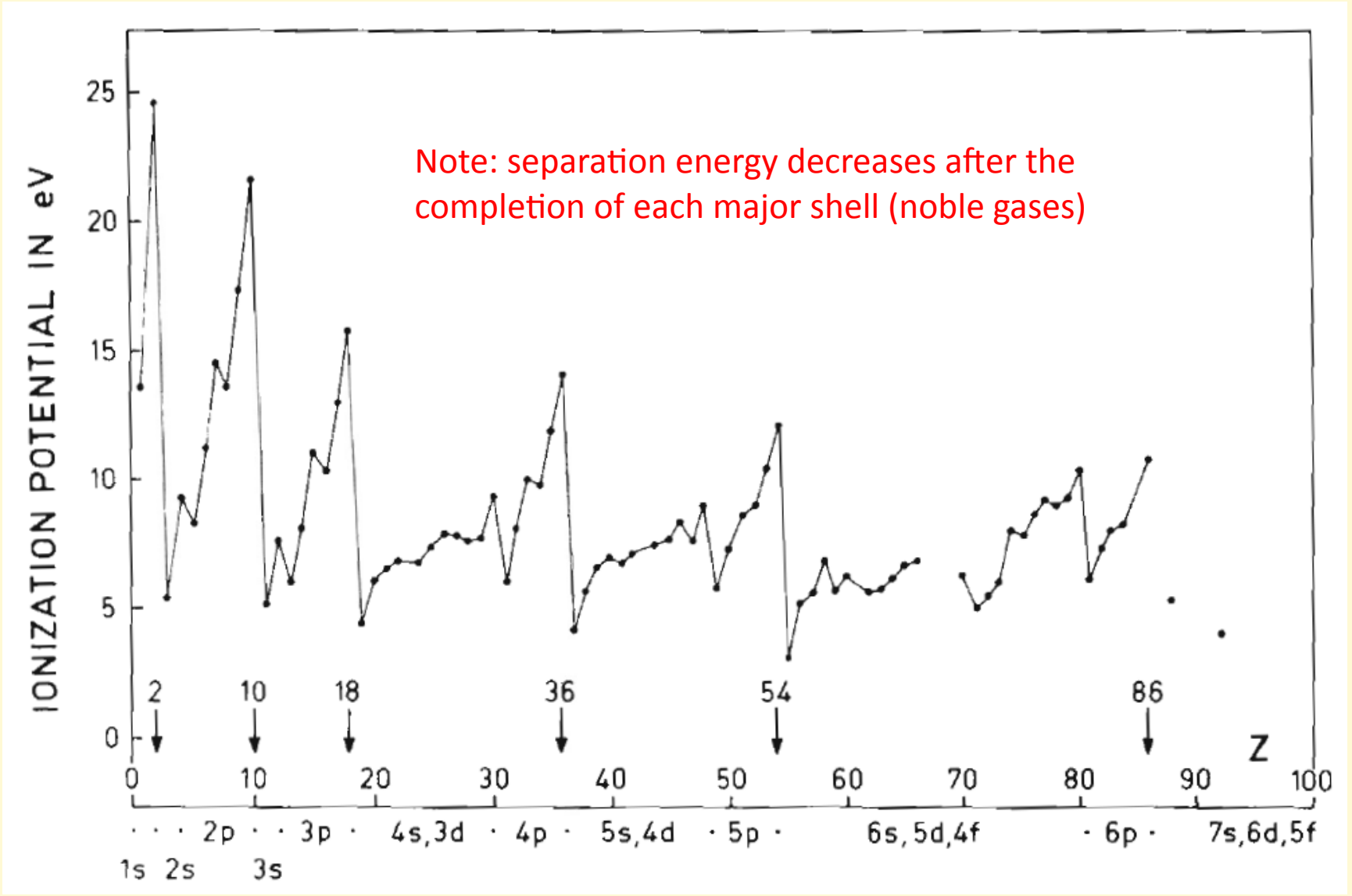
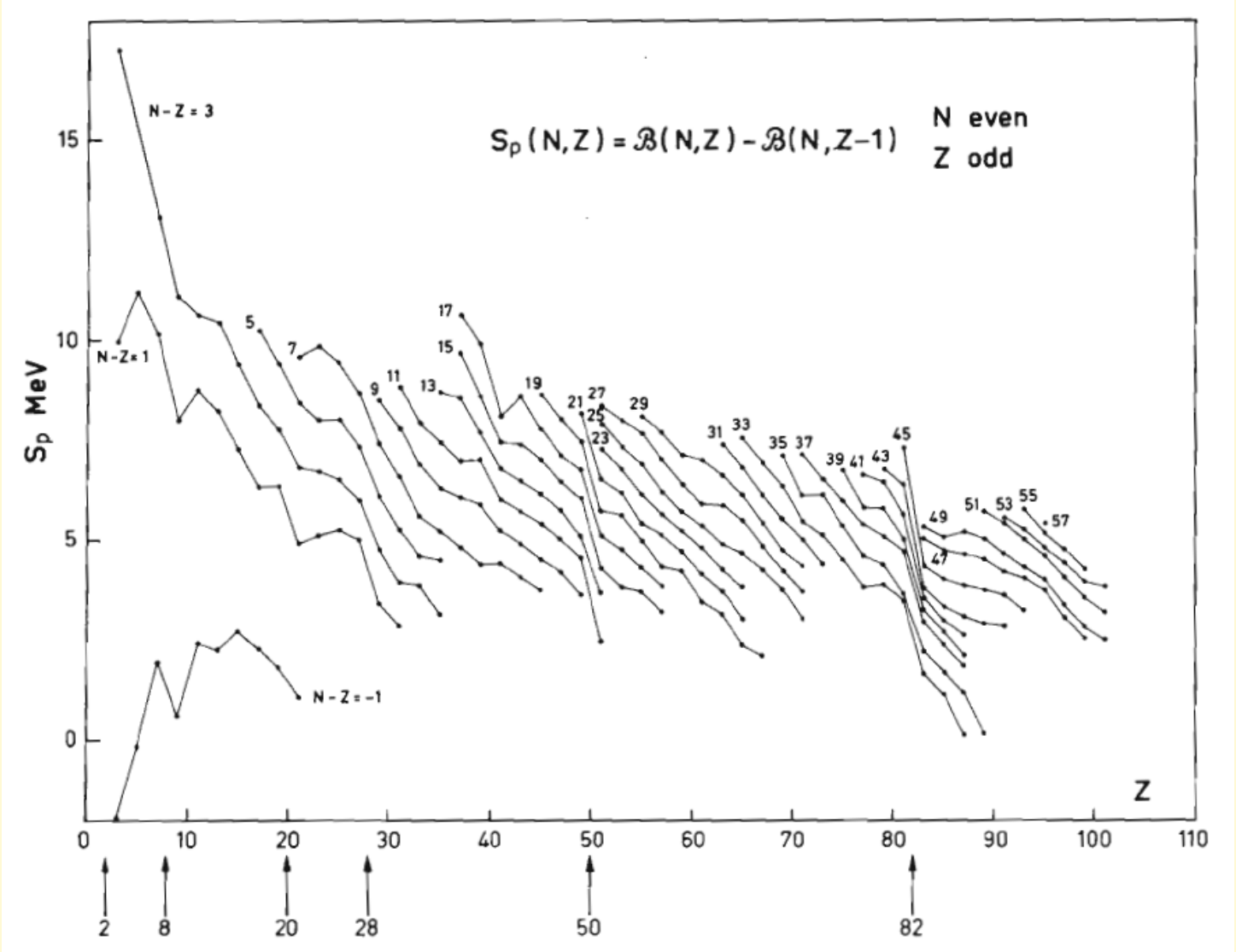
Separation energy of last electron for neutral atoms

Ref: C.E. Moore, *Atomic Energy Levels*, Circular 467, vol. 1, p .XL,
Nat. Bureau of Standards, Washington, D.C. (1949)



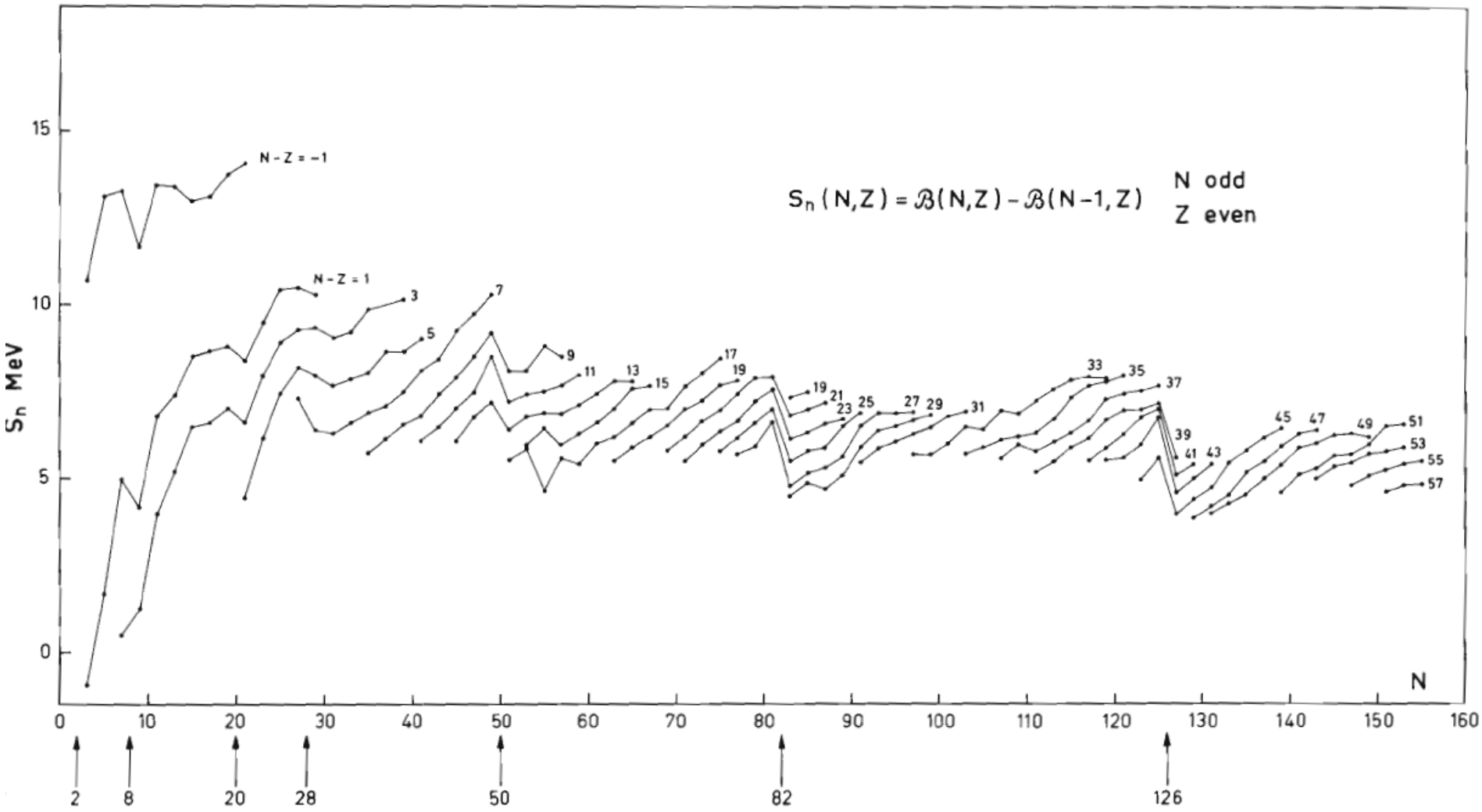
One-proton separation energy for nuclei

Ref: J.H.E. Mattauch, W. Thiele, and A.H. Wapstra, Nucl. Phys. 67 (1965) 1

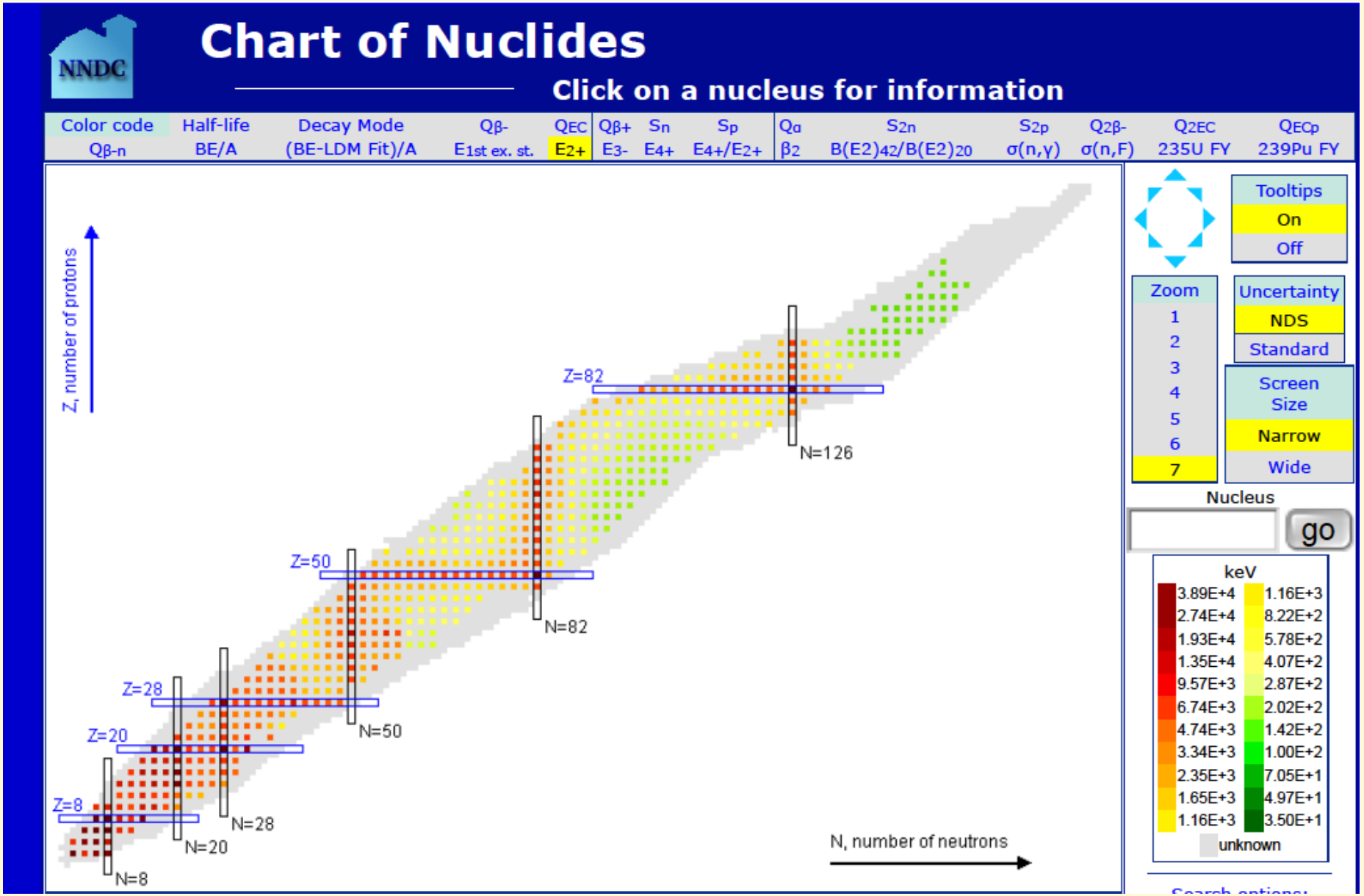


One-neutron separation energy for nuclei

Ref: J.H.E. Mattauch, W. Thiele, and A.H. Wapstra, Nucl. Phys. 67 (1965) 1



Energies of first 2^+ states in even-even nuclei



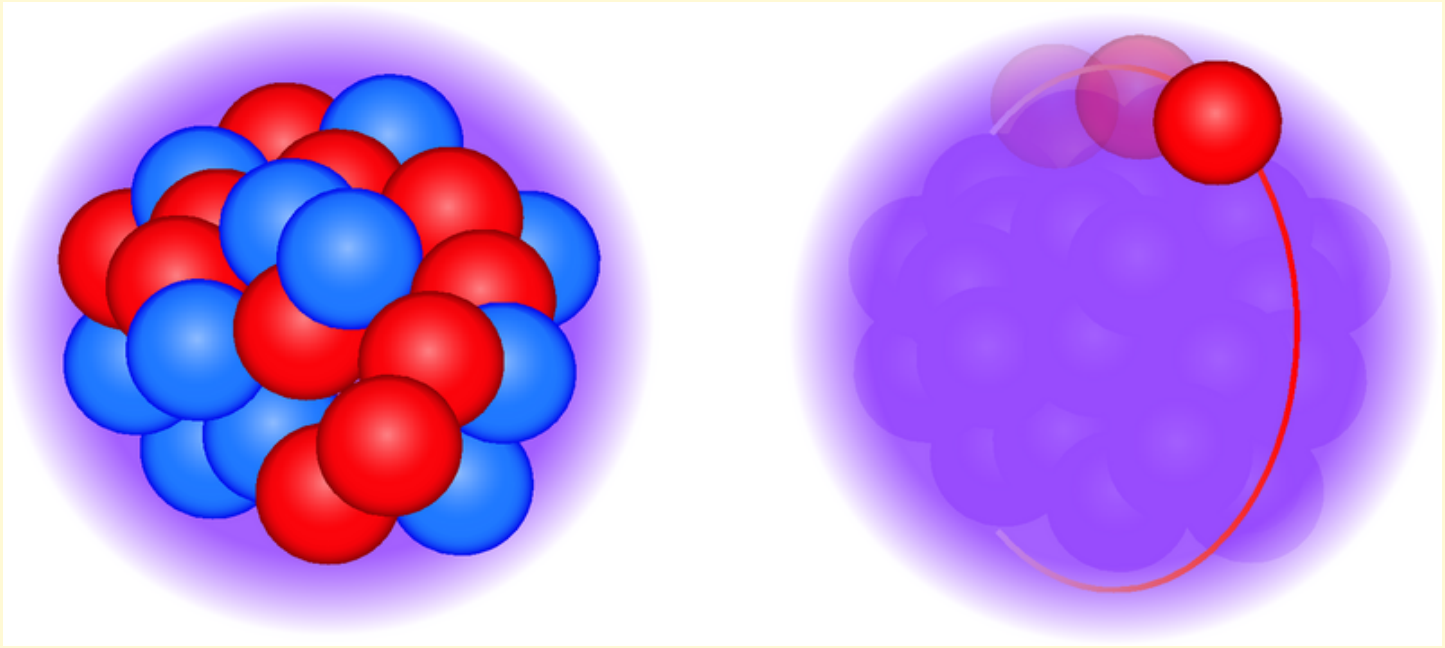
Spherical shell model with strong spin-orbit force (1949)

Maria Goeppert Mayer, U.S.A. J. Hans D. Jensen, Germany

Nobel prize 1963

Mean-field concept

(Ref: Isotope Science Facility at Michigan State University,
MSUCL-1345, p. 41, Nov. 2006)



Spherical shell model with spin-orbit interaction: quantum numbers

$n = 1,2,3,...$

$\ell = 0,1,2,...$

$N = 2(n-1) + \ell$ determines 3D harmonic oscillator energy

$j = |\ell - 1/2|$ and $j = \ell + 1/2$

$E_{n\ell j} = -V_0 + \hbar\omega(N + 3/2) - \alpha\ell$ for $j = \ell + 1/2$

$E_{n\ell j} = -V_0 + \hbar\omega(N + 3/2) + \alpha(\ell + 1)$ for $j = |\ell - 1/2|$

Spherical shell model with spin-orbit interaction: single-particle quantum states

ℓ =	0	1	2	3	4	5	6
	s	p	d	f	g	h	i

spectroscopic notation

$n\ell_j$ notation for single particle energy states, e.g.

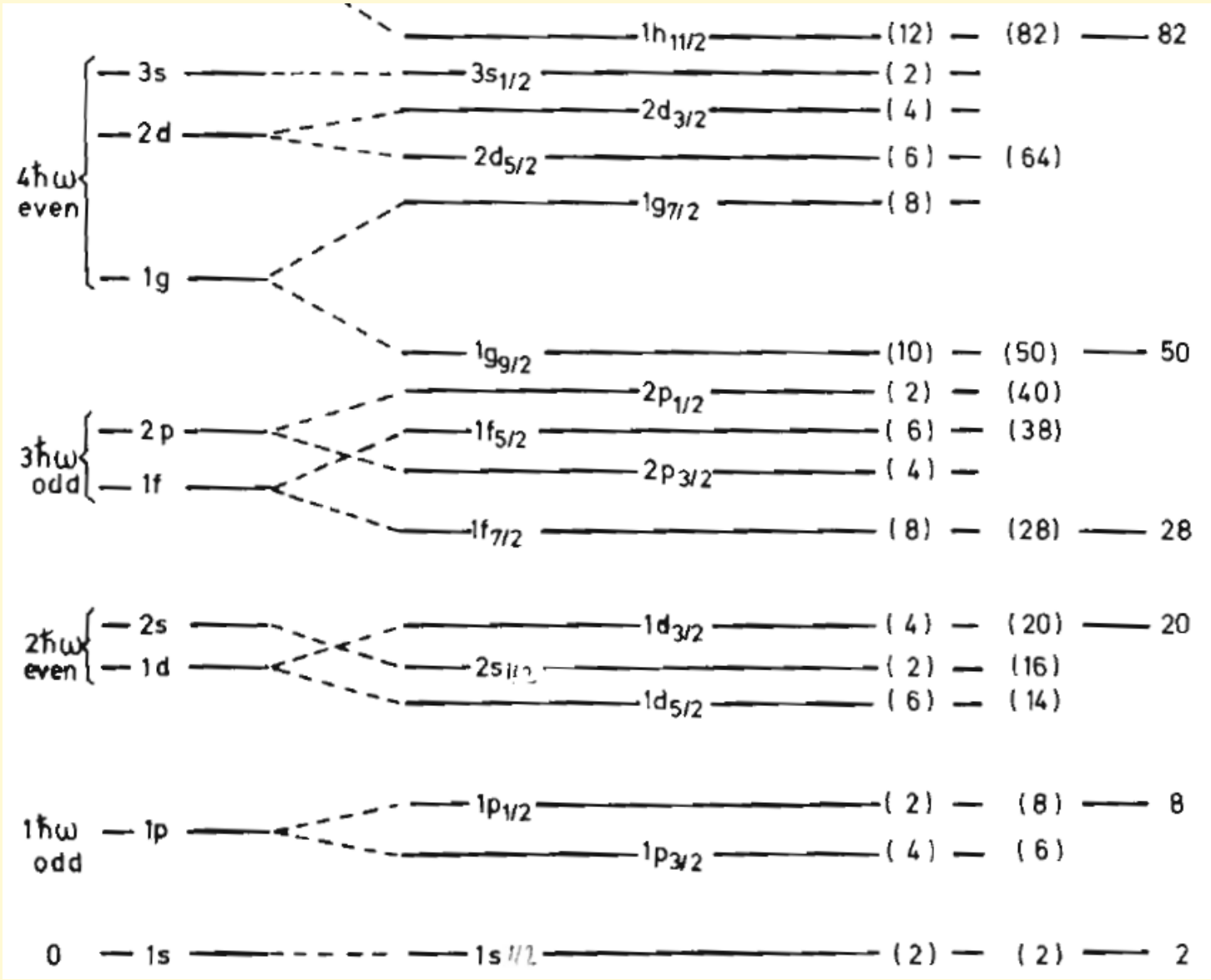
$$1s_{1/2} \quad 1p_{3/2} \quad 2d_{5/2} \quad 1h_{11/2} \quad 1i_{13/2}$$

$\left|n\ell j m_j\right\rangle$ single particle state vectors

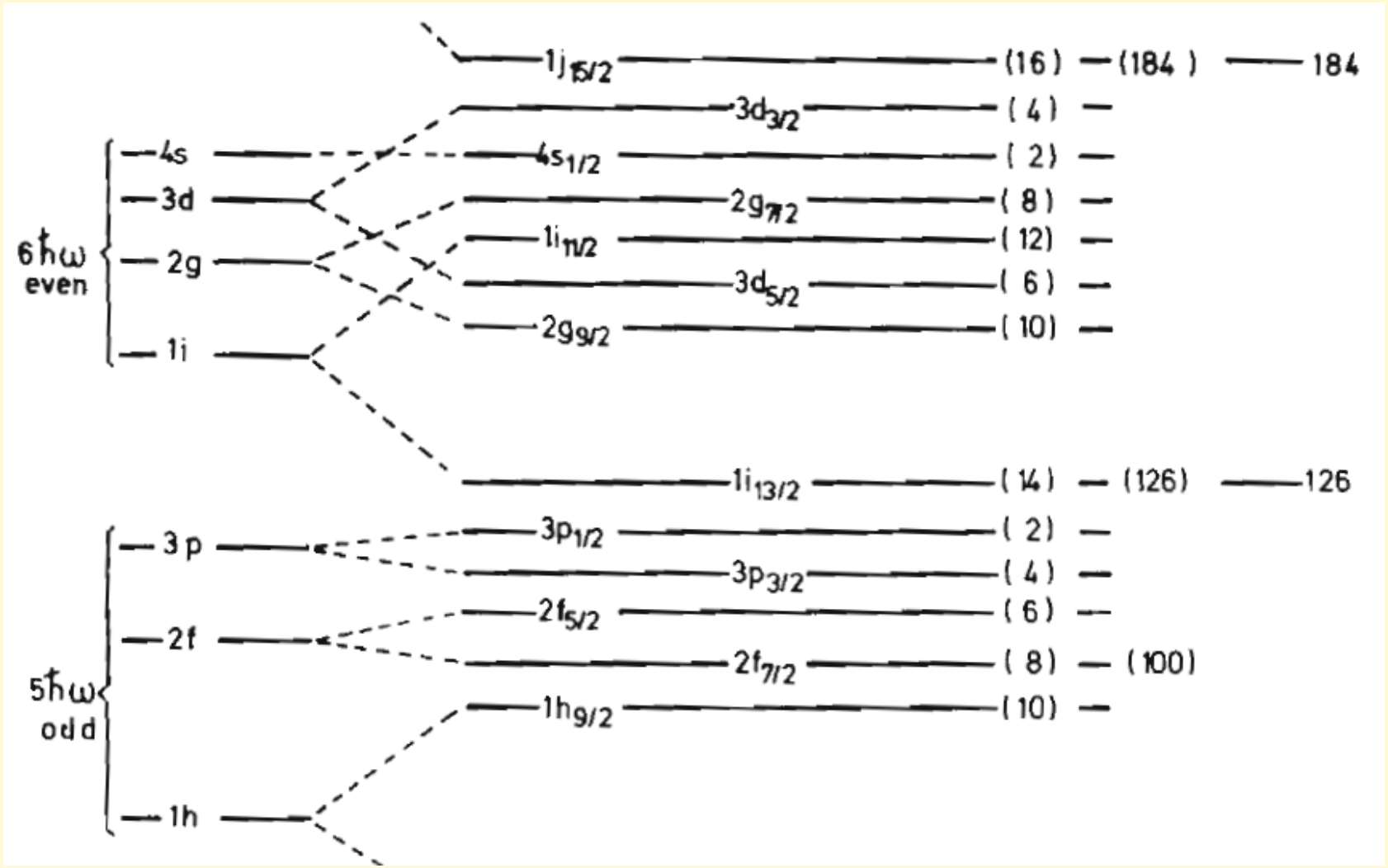
$m_j = -j, -j+1, \ldots, +j$

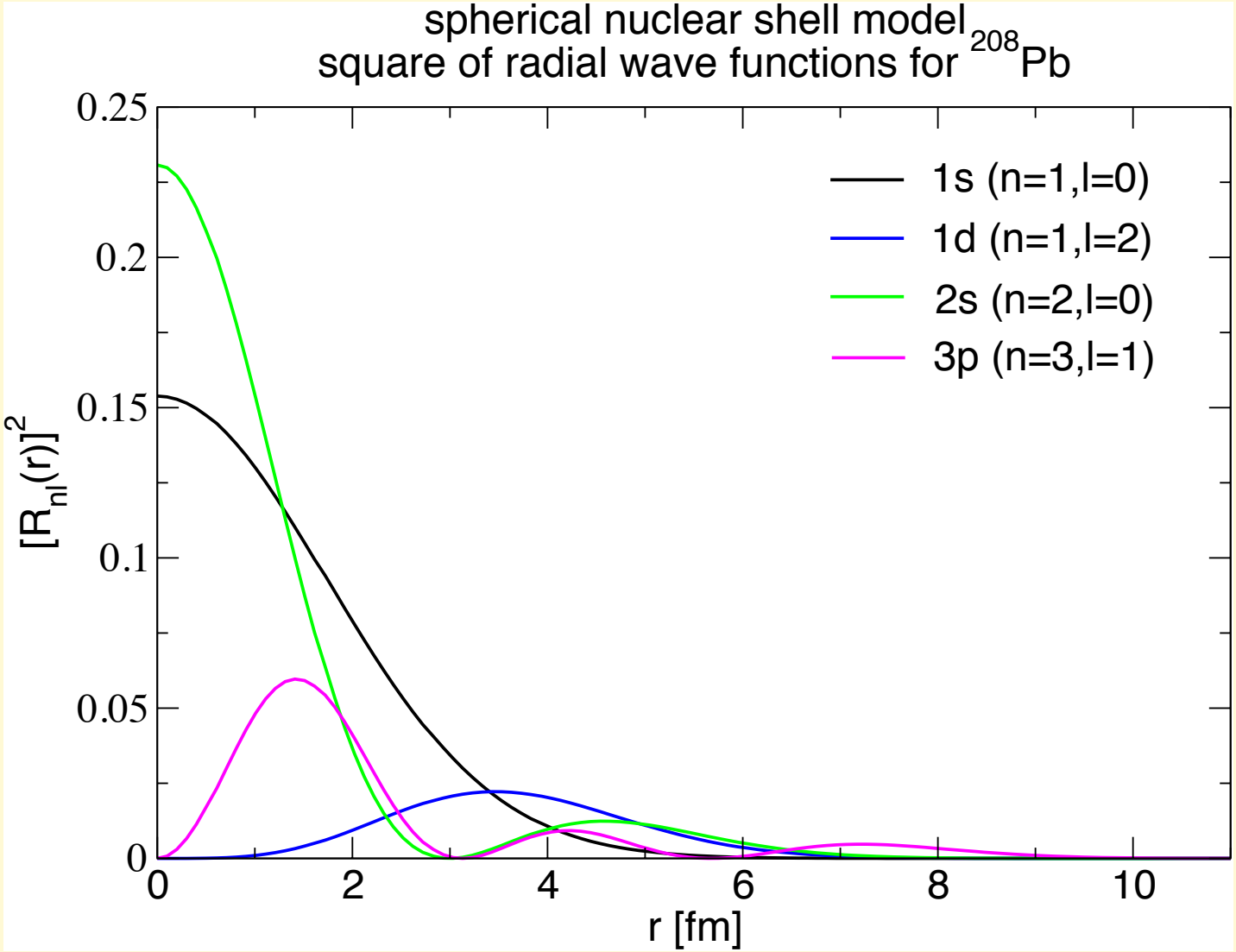
$\Rightarrow M = 2j+1$ degeneracy of single-particle energy level

Spherical shell model (Mayer & Jensen, 1955): part 1

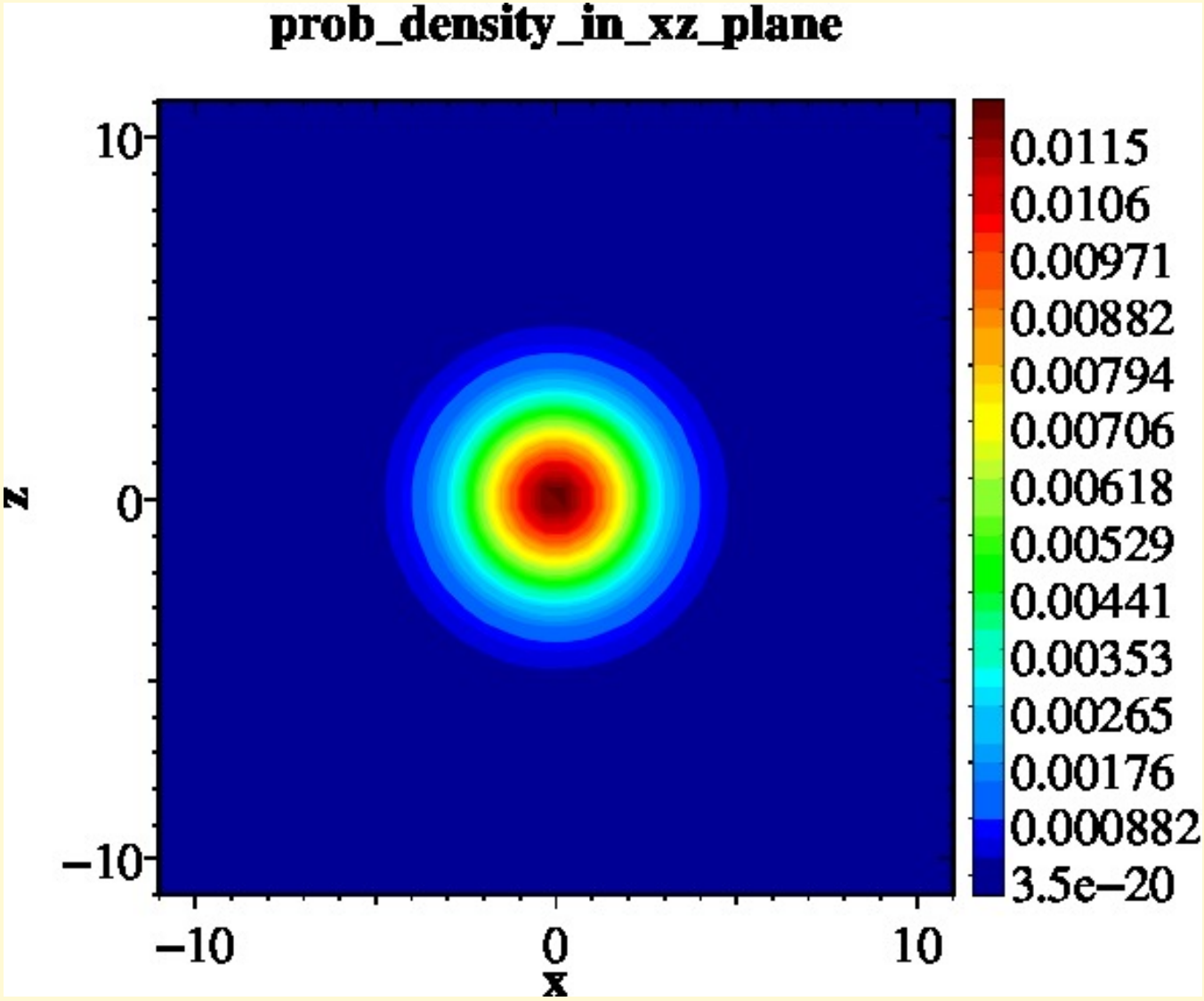


Spherical shell model (Mayer & Jensen, 1955): part 2

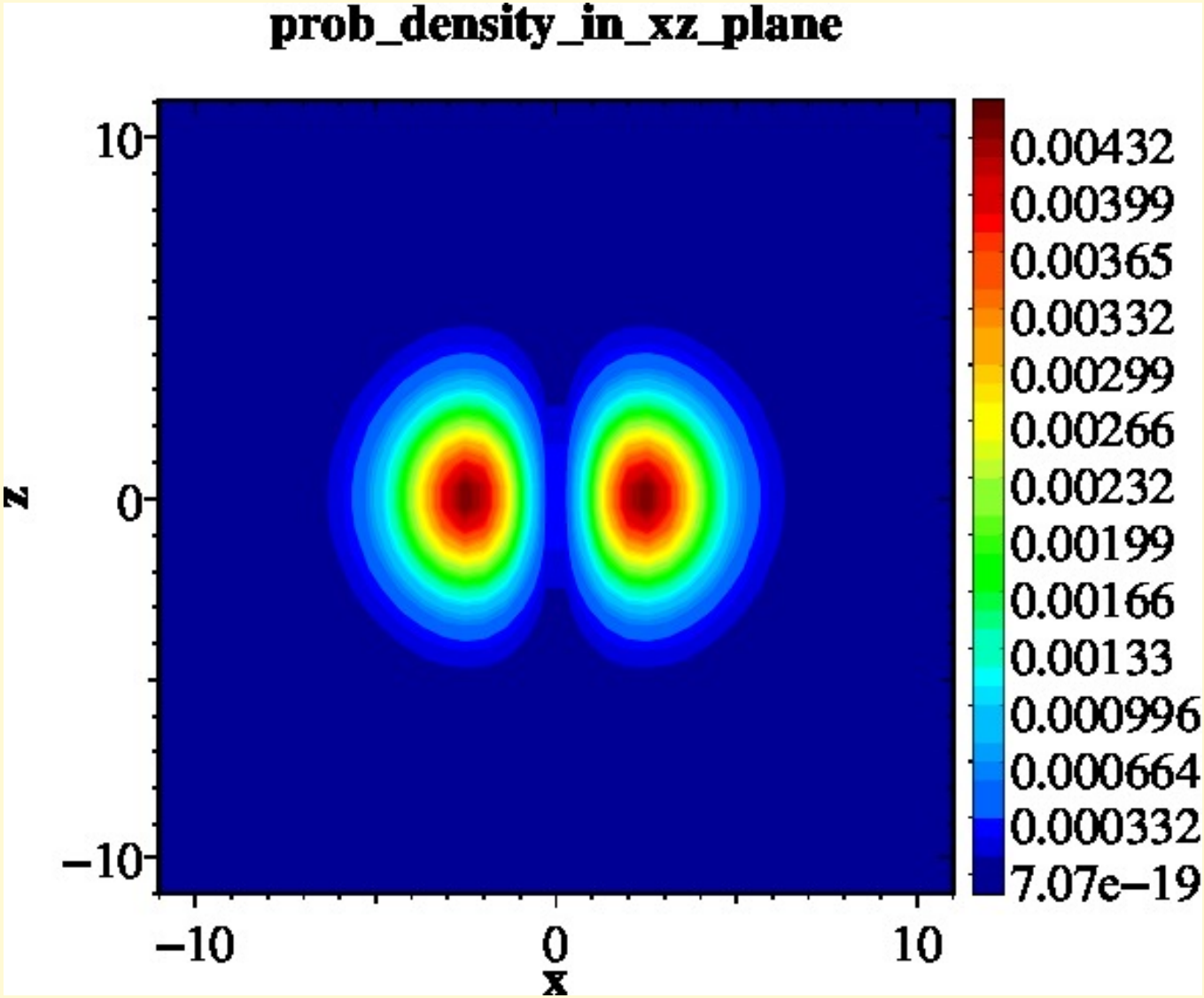



spherical nuclear shell model
square of radial wave functions for ^{208}Pb
$[R_{nl}(r)]^2$
r [fm]
1s (n=1,l=0)
1d (n=1,l=2)
2s (n=2,l=0)
3p (n=3,l=1)
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10

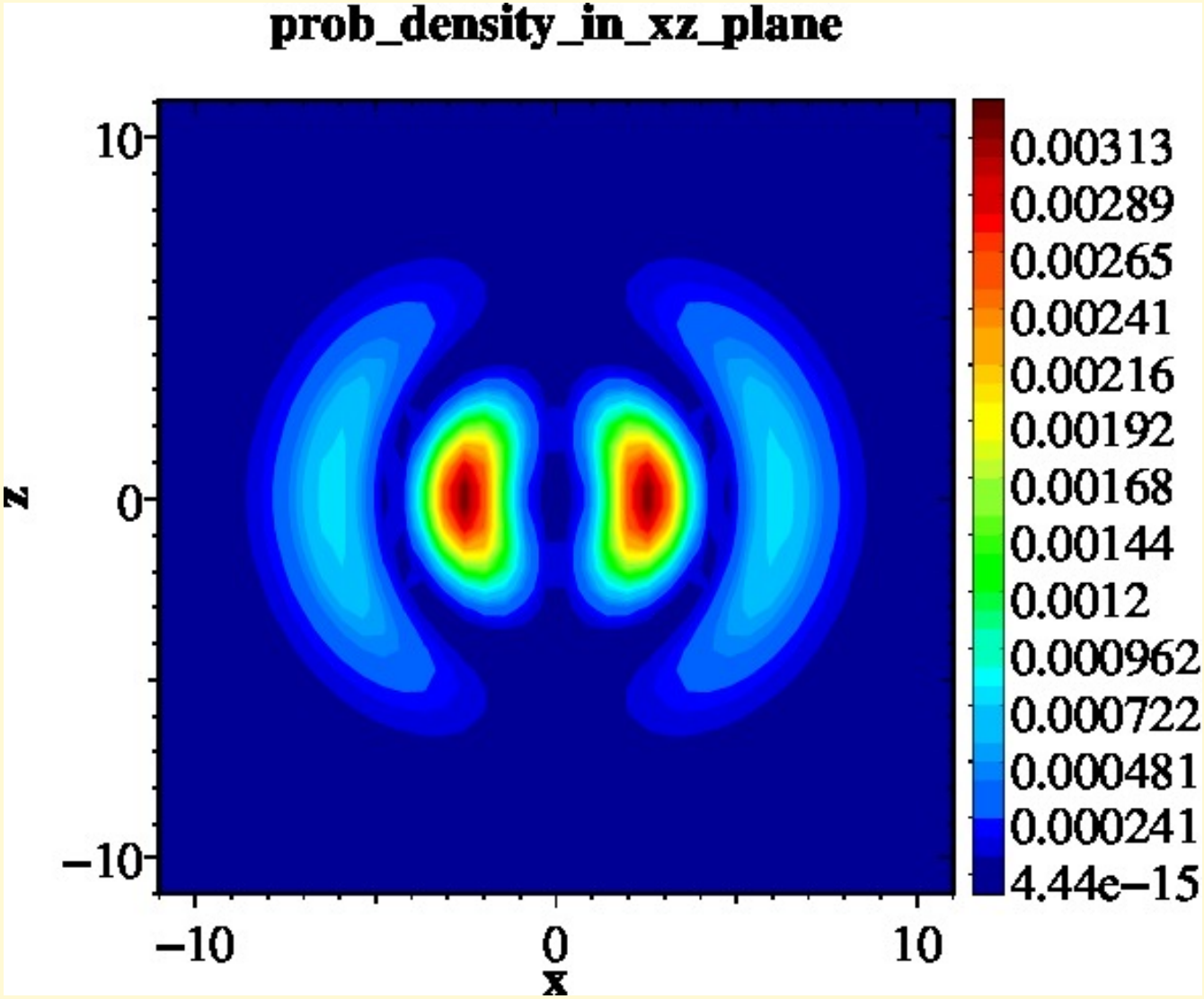
Spherical shell model for ^{208}Pb: $1s_{1/2}$ orbital ($m_j = +1/2$)



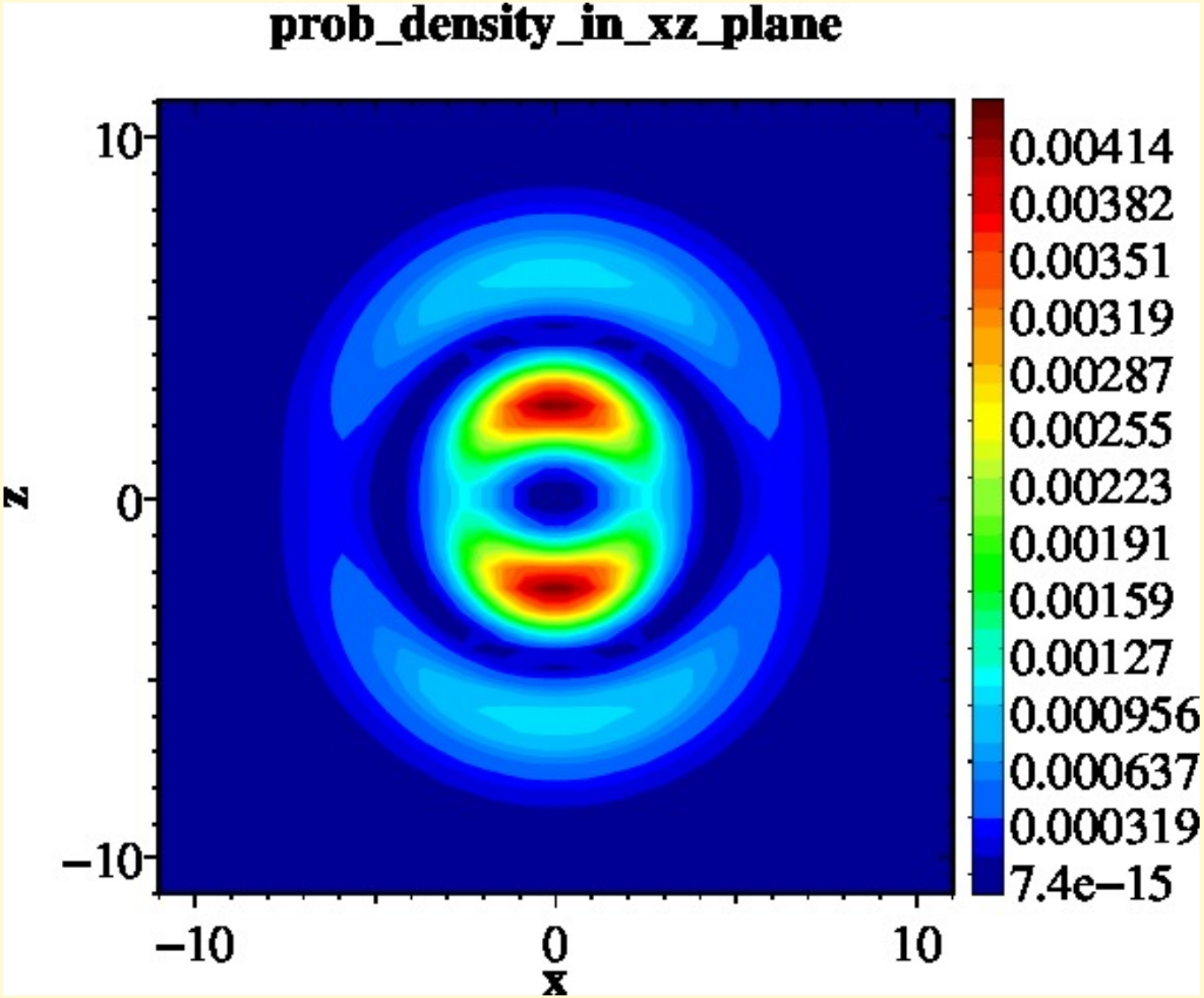
Spherical shell model for ^{208}Pb: $1p_{3/2}$ orbital (m_j = +3/2)



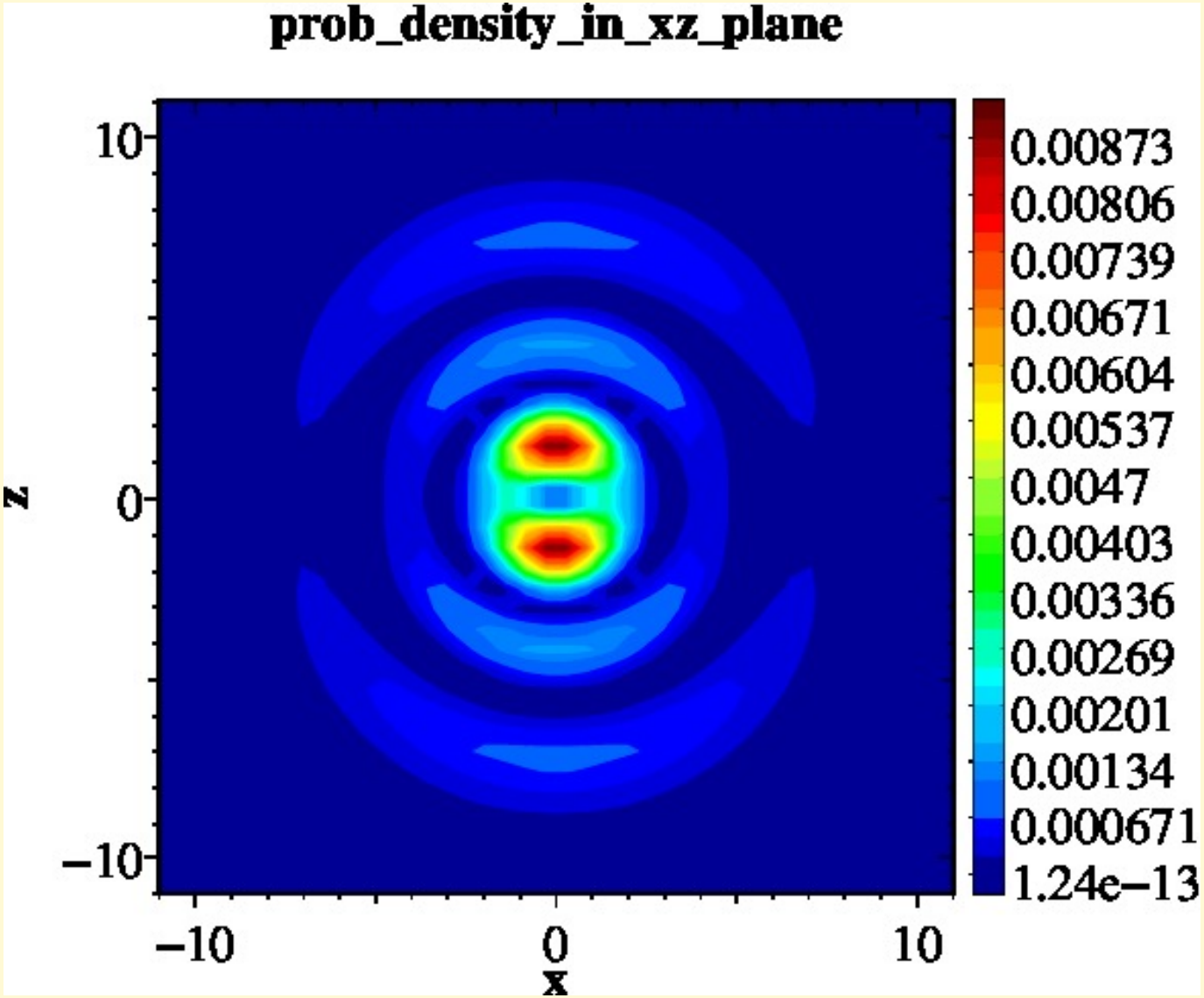
Spherical shell model for ^{208}Pb: $2d_{3/2}$ orbital (m_j = +3/2)



Spherical shell model for ^{208}Pb: $2d_{3/2}$ orbital ($m_j = +1/2$)



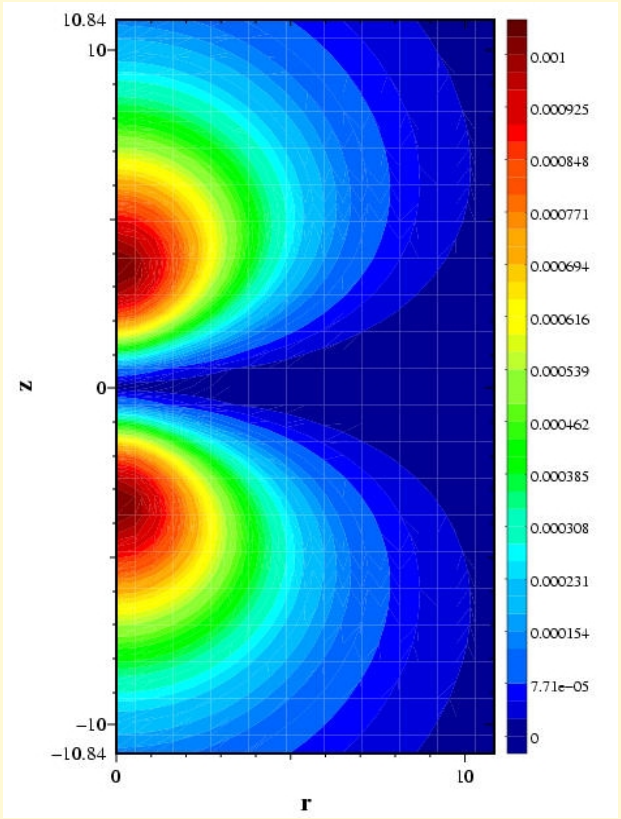
Spherical shell model for ^{208}Pb: $3p_{3/2}$ orbital (m_j = +1/2)



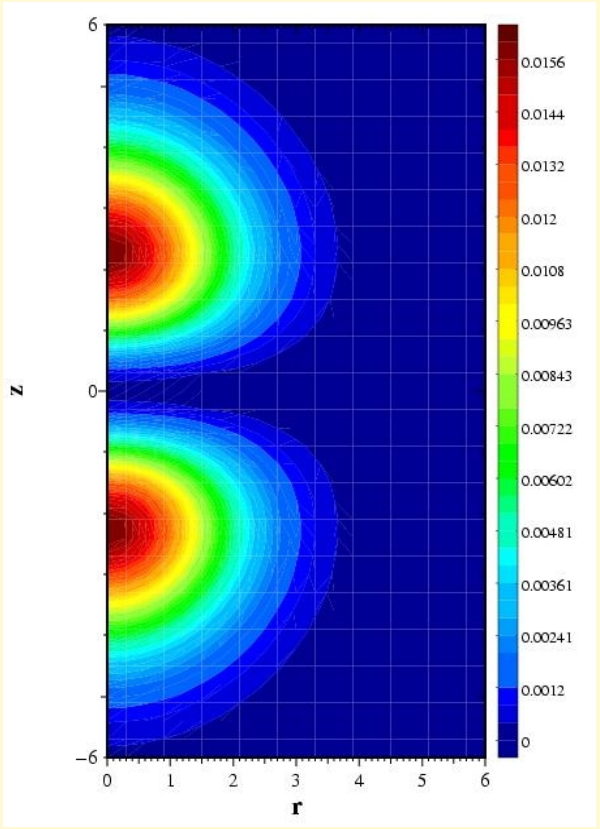
Comparison of one-particle shell models

atomic “orbitals” (left) vs. nuclear “orbitals” (right)

Note that atomic orbitals are more diffuse!



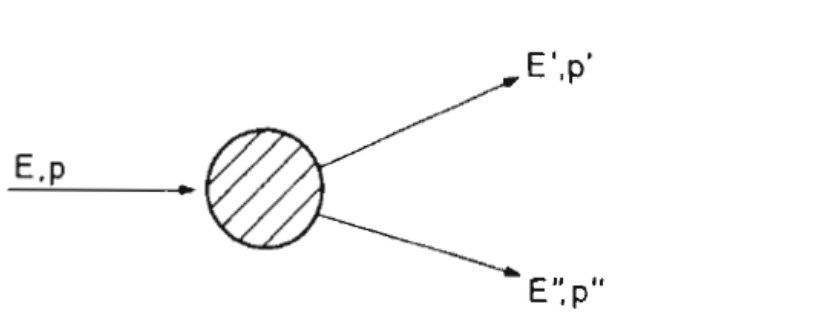
size: 10^{-10} m



size: 10^{-15} m

Measurement of single-particle proton energy levels: $^{16}O(p,2p)^{15}N$

Ref: Textbook by Ring & Schuck



Schematic representation of the $(p, 2p)$ reaction.

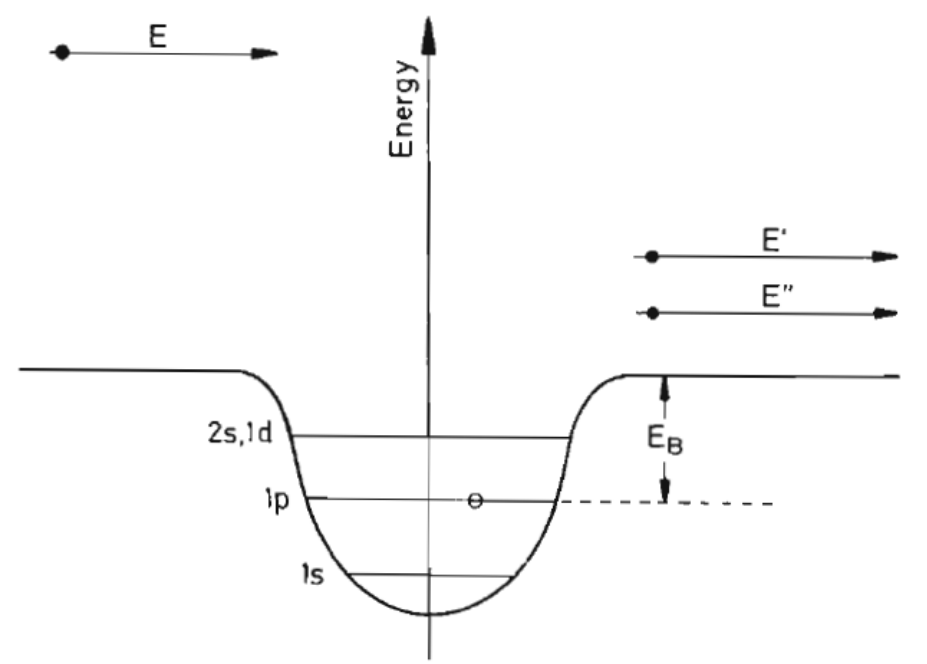


Figure 2.12. $(p, 2p)$ reaction in the shell model.

Measured single-particle proton energy levels in ^{16}O

Ref: Textbook by Ring & Schuck

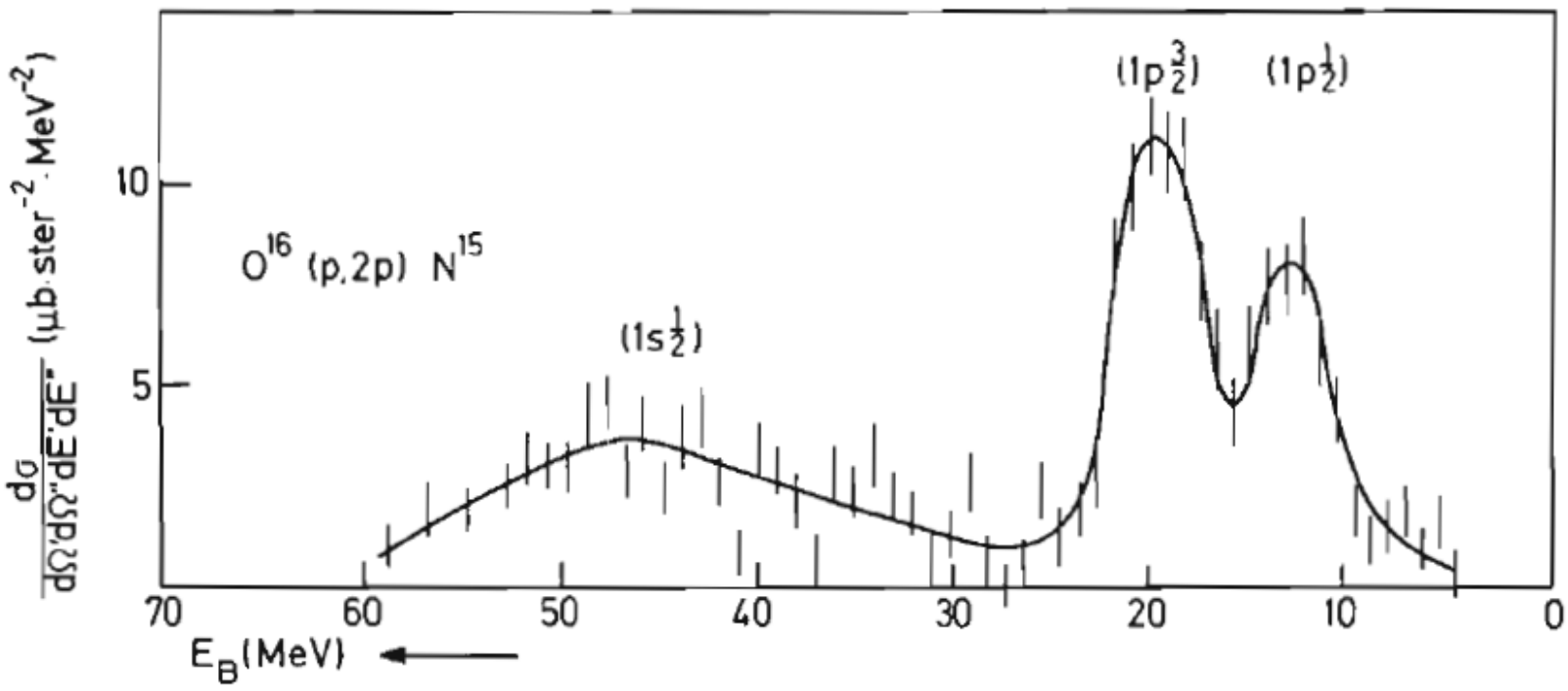


Figure 2.13. Experimental ^{16}O $(p, 2p)$ ^{15}N cross section [MHT 58].

Measured single-particle neutron energy levels in ^{209}Pb

Ref: Ellegaard et al., Nucl. Phys. A129 (1969) 113

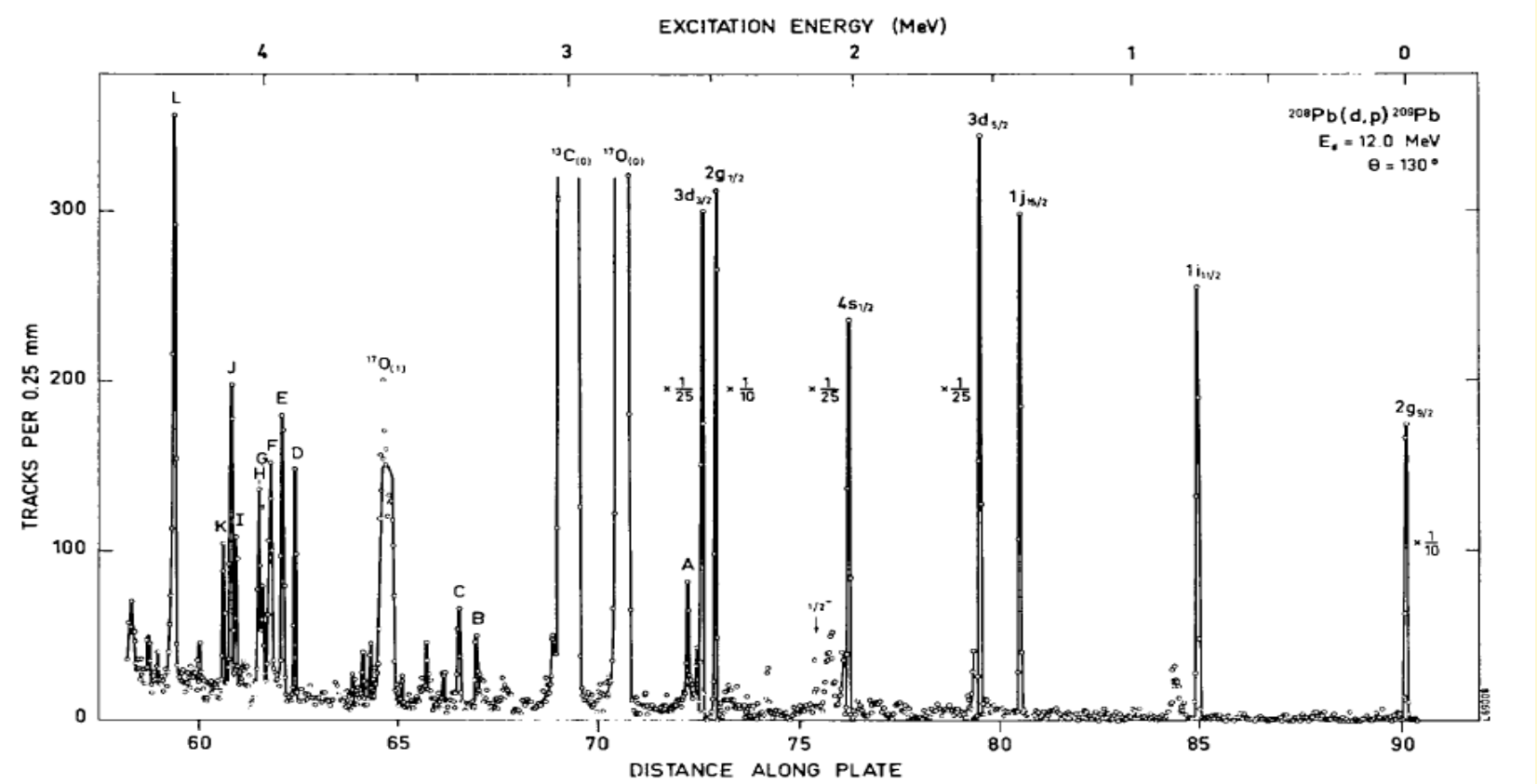


Fig. 6. Proton spectrum from the ^{208}Pb(d, p)^{209}Pb reaction. The intense peaks are labelled by their shell-model assignment. The peaks labelled A . . . L are the peaks for which angular distributions are measured. The arrow labelled $\frac{1}{2}^-$ indicates the position of the known $\frac{1}{2}^-$ level.