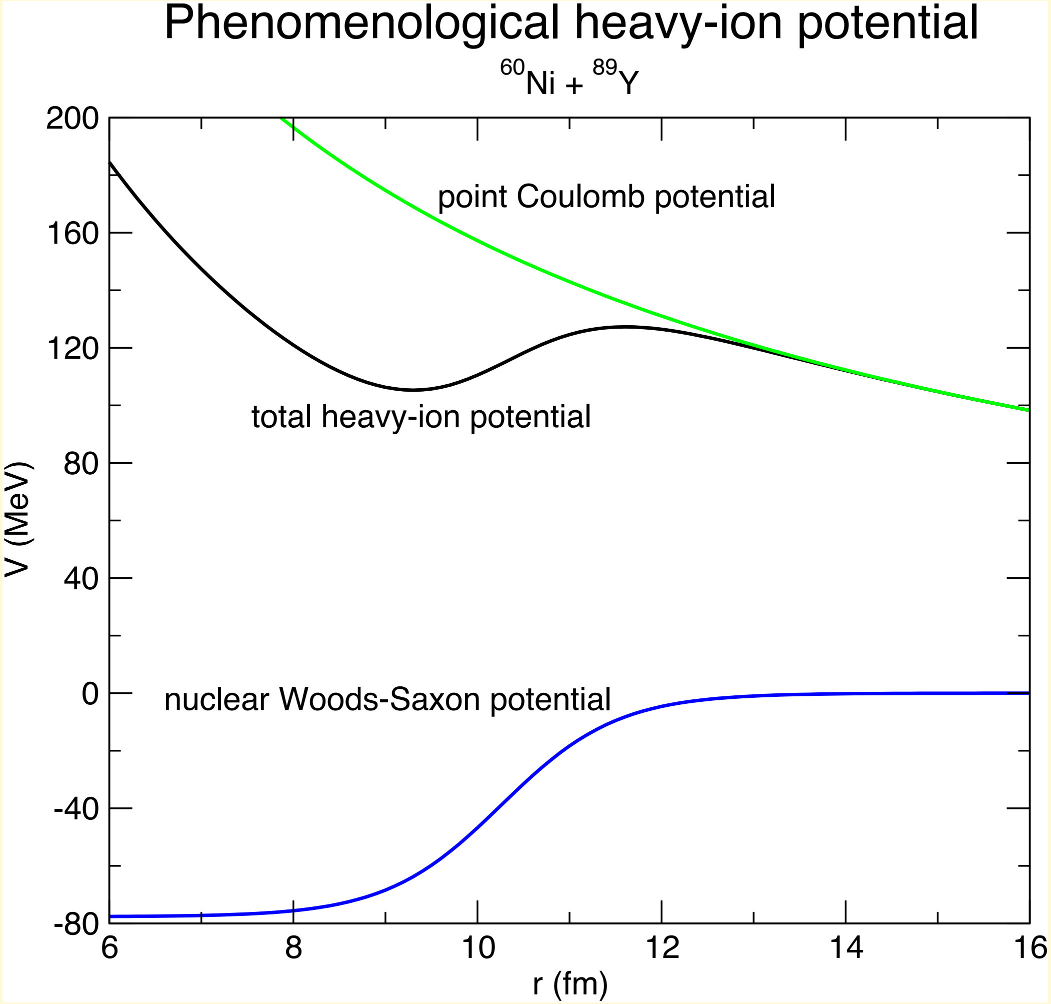
Introduction: heavy-ion potential model for sub-barrier fusion calculations



$$V(r) = V_{Coul}(r) + V_{nucl}(r)$$

Introduction: WKB barrier transmission probability

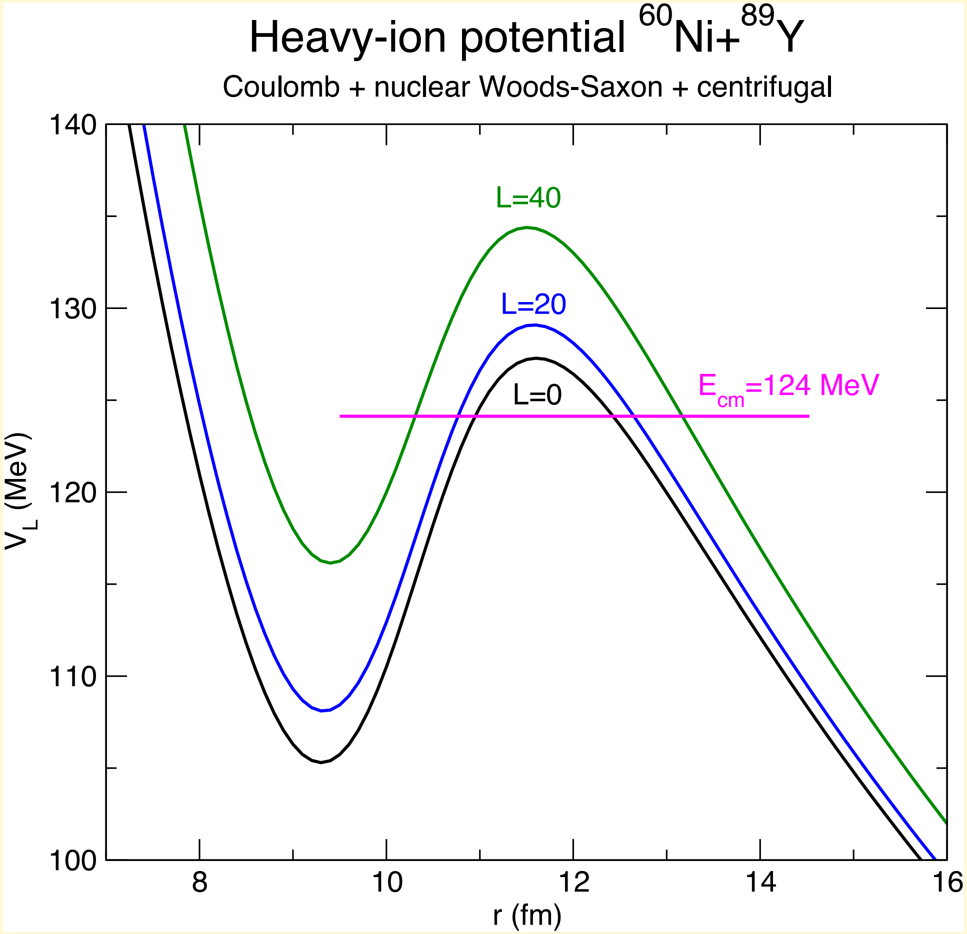
$$V_\ell(r) = V(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}$$

WKB barrier transmission probability
(see e.g. Shankar, QM, p. 444)

$$T_\ell(E_{cm}) = \exp\left[-2S_\ell(E_{cm})\right]$$

where the WKB barrier penetration integral is given by

$$S_\ell(E_{cm}) = \frac{1}{\hbar}\int_{r_1(\ell)}^{r_2(\ell)} dr \sqrt{2\mu[V_\ell(r) - E_{cm}]}$$



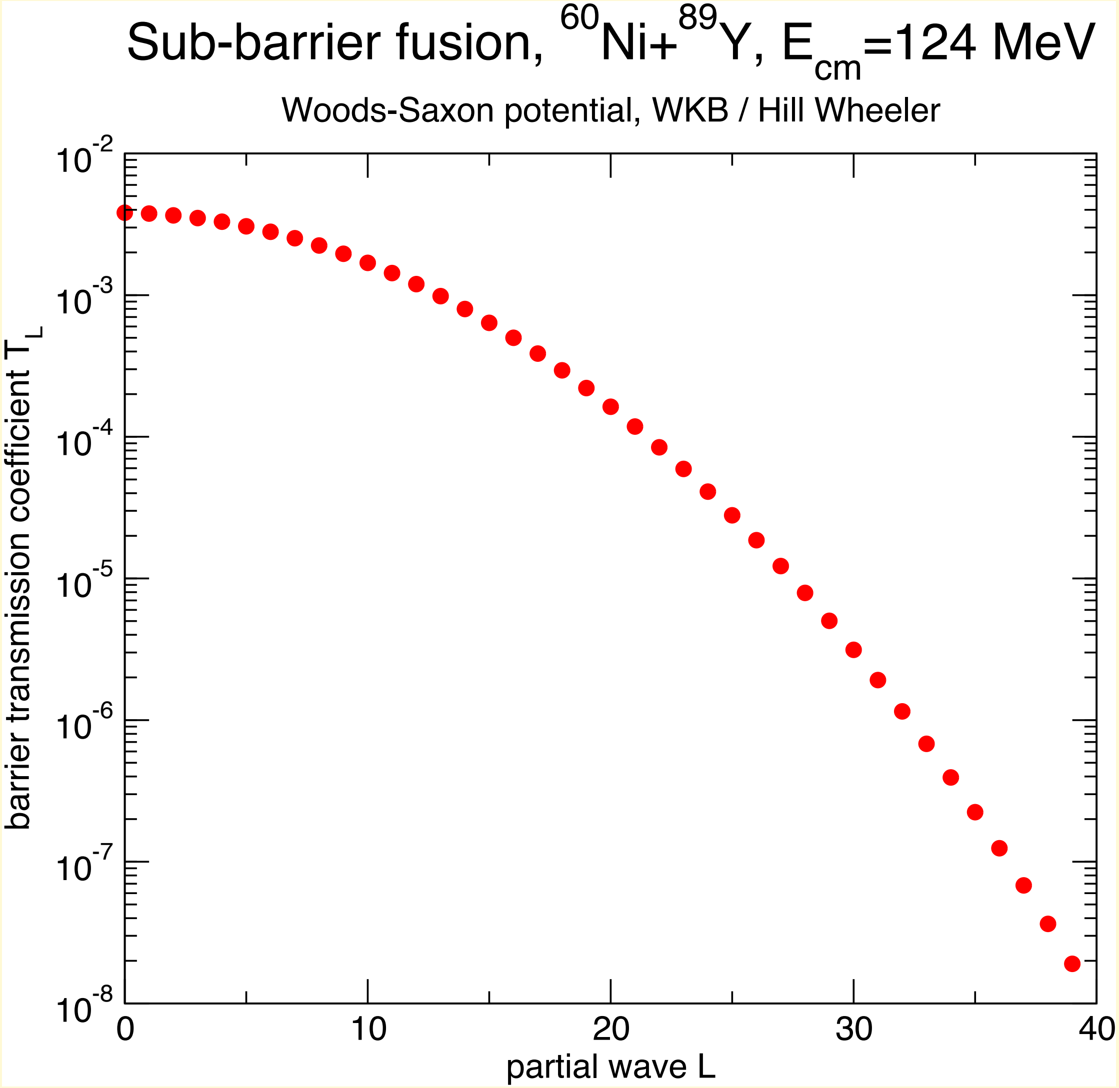
Introduction: Hill-Wheeler extension of WKB method

barrier transmission probability (Hill-Wheeler extension of WKB)

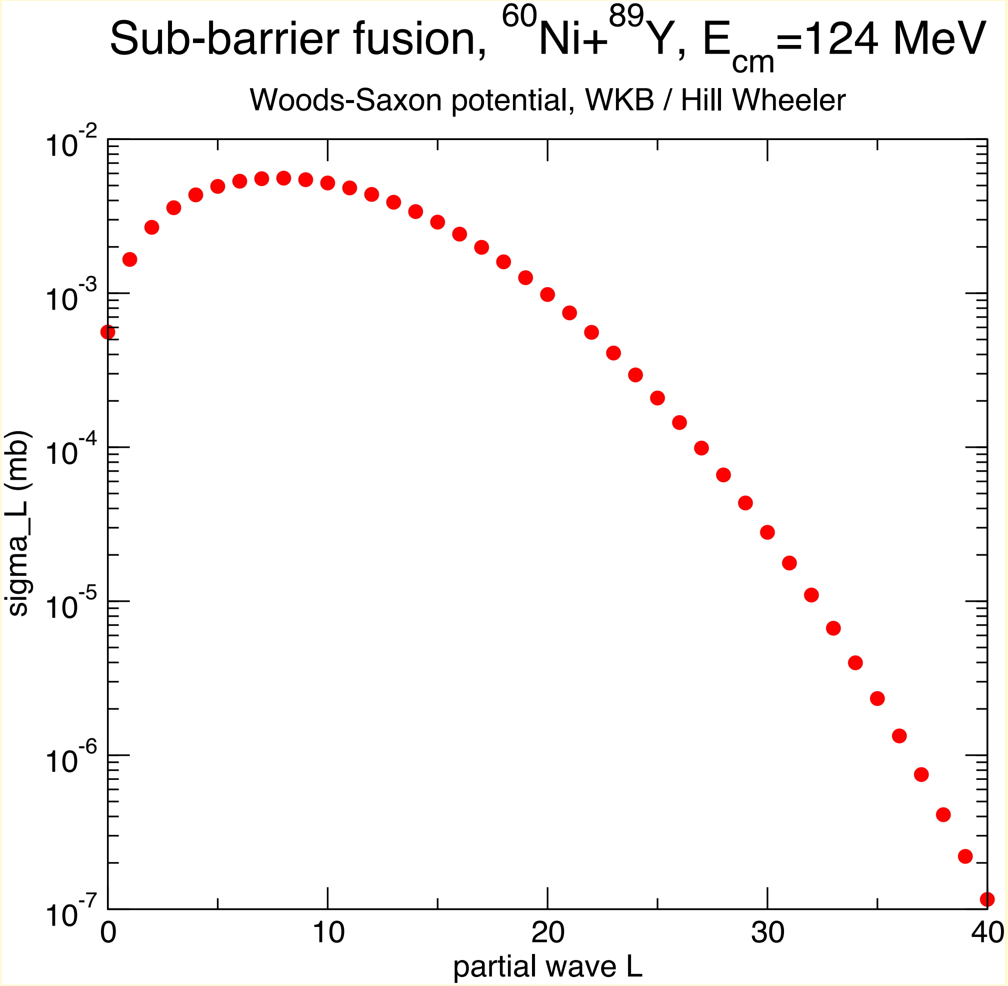
$$T_\ell(E_{cm}) = \frac{1}{1+\exp\left[2S_\ell(E_{cm})\right]}$$

In the limit $\exp(2S_\ell) >> 1$

we recover the standard WKB result.



Introduction: partial and total fusion cross section



partial fusion cross section

$$\sigma_\ell(E_{cm}) = \frac{\pi\hbar^2}{2\mu E_{cm}}(2\ell+1)T_\ell(E_{cm})$$

total fusion cross section

$$\sigma_{tot}(E_{cm}) = \sum_{\ell=0}^{\infty}\sigma_\ell(E_{cm})$$

Theoretical description of heavy-ion fusion at sub-barrier energies

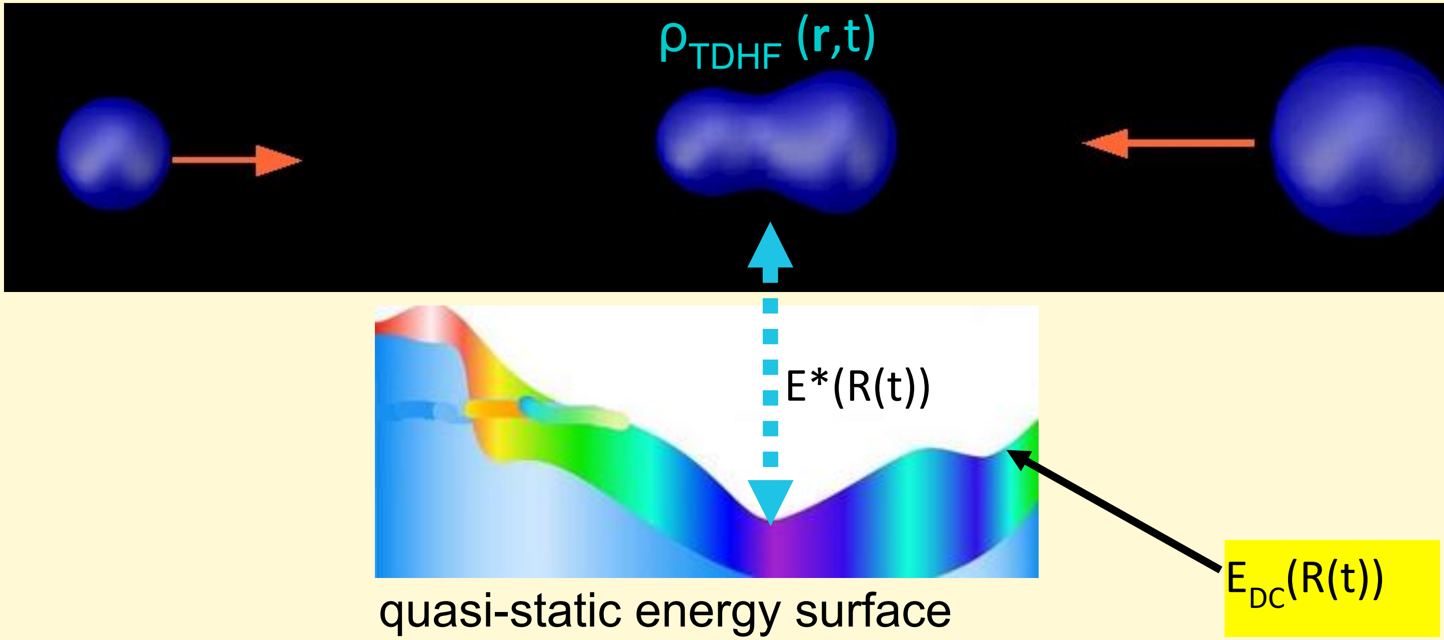
- Barrier penetration models (phenomenological heavy-ion potentials)
  Balantekin and Takigawa, Rev. Mod. Phys. 70, 77 (1998)
- Coupled channels calculations (phenomenological heavy-ion potentials)
  S. Misicu and H. Esbensen, PRL 96, 112701 (2006)
  Ichikawa, Hagino, and Iwamoto, PRC 75, 057603 (2007)
- TDHF with density-constraint: microscopic calculation of heavy-ion potentials + barrier tunneling
  Umar and Oberacker, Phys. Rev. C 74, 021601(R) (2006)
  Umar and Oberacker, Phys. Rev. C 76, 014614 (2007)
  Umar, Oberacker, Maruhn, and Reinhard, Phys. Rev. C 81, 064607 (2010)
  Oberacker, Umar, Maruhn, and Reinhard, Phys. Rev. C 85, 034609 (2012)

Density-constrained TDHF calculations

- microscopic calculation of heavy-ion potentials V(R) and coordinate-dependent mass parameters M(R)
- barrier tunneling:
  exact numerical solution of Schrödinger eq. for relative coordinate R with "Incoming Wave Boundary Condition"
  Ref: Hagino, Rowley, and Kruppa, Comput. Phys. Commun. 123, 143 (1999)
- calculate fusion cross sections below and above the barrier

TDHF with Density-Constraint (DC-TDHF)

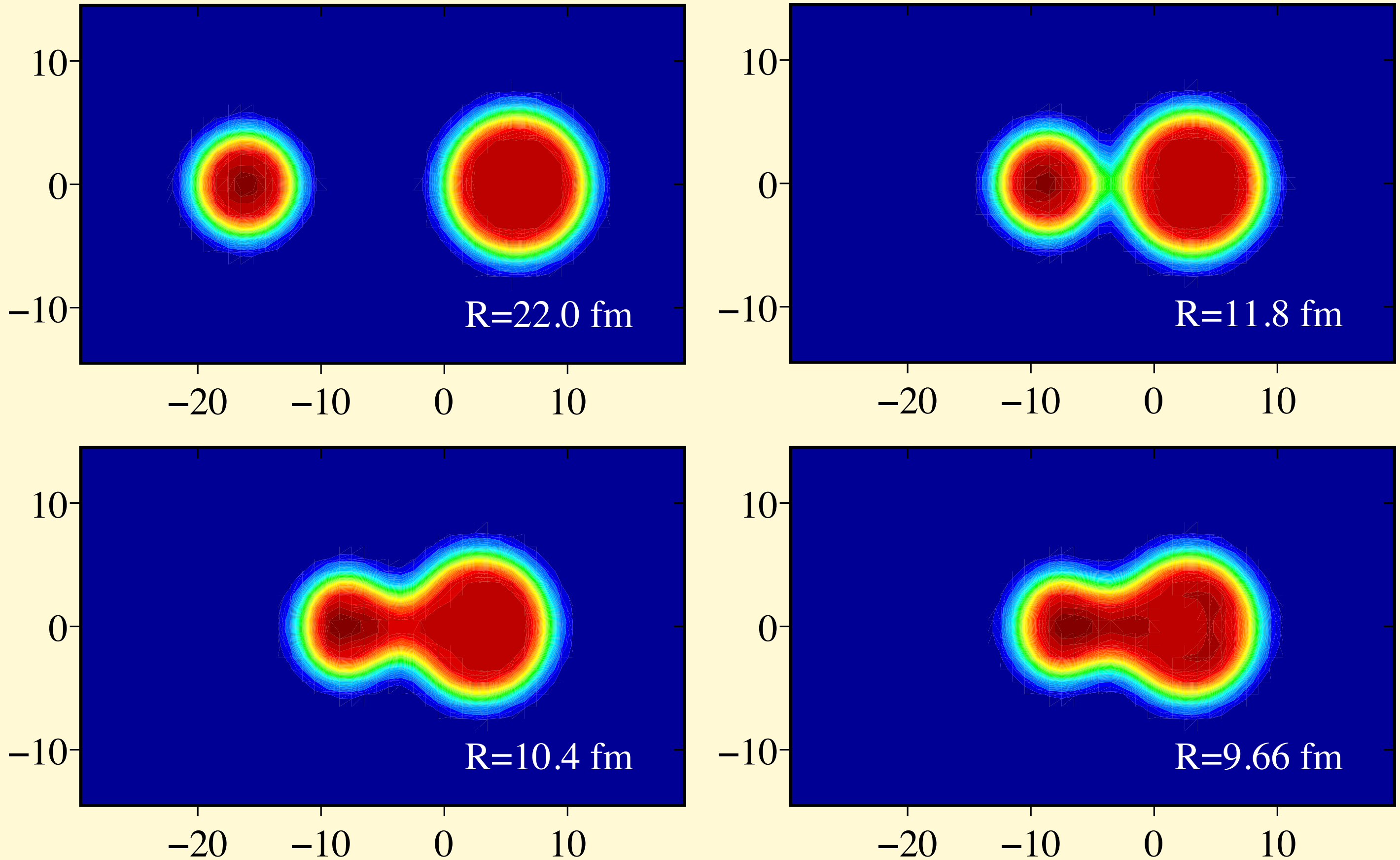
- A method to extract internal excitation energy while holding the <u>instantaneous</u> neutron and proton densities constrained.
- TDHF provides the dynamical densities for calculating V(R)



$$\rightarrow \quad V(R) = E_{\mathrm{DC}}(R) - E_{\mathrm{A}_1} - E_{\mathrm{A}_2}$$

Calculation of heavy-ion potential with DC-TDHF method

$$V(R) = E_{\mathrm{DC}}(R) - E_{\mathrm{A}_1} - E_{\mathrm{A}_2}$$

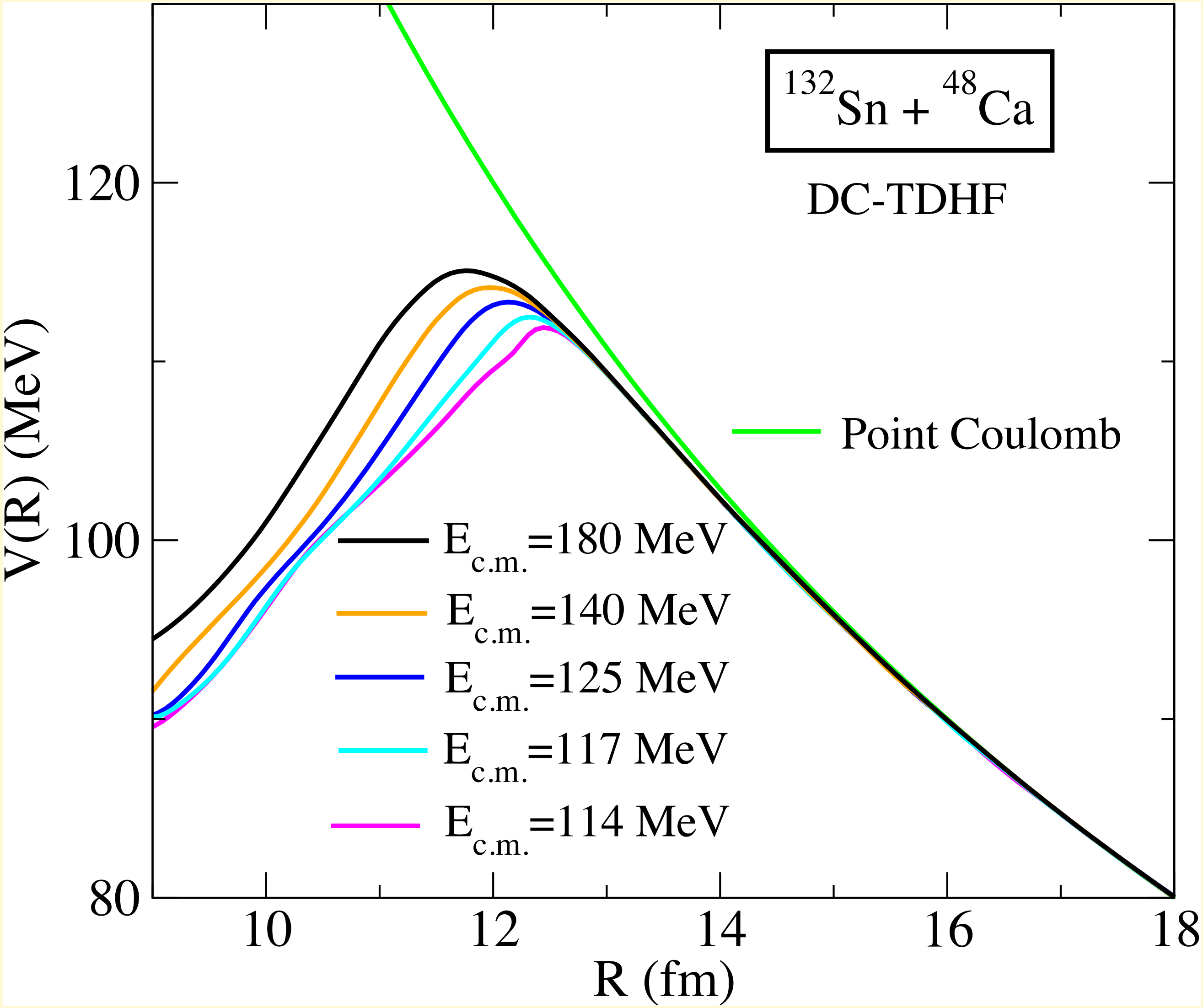


$^{48}Ca+^{132}Sn$
E_{cm}=140 MeV

V(R) contains dynamical entrance channel effects (neck formation, particle transfer, surface vibrations, giant resonances)

Heavy-ion potential: strong E_{cm}-dependence

Oberacker, Umar, Maruhn, and Reinhard, Phys. Rev. C 85, 034609 (2012)



Dynamical Effective Mass from TDHF

$$E_{\mathrm{c.m.}} = \frac{1}{2} M(R) \dot{R}^2 + V(R)$$

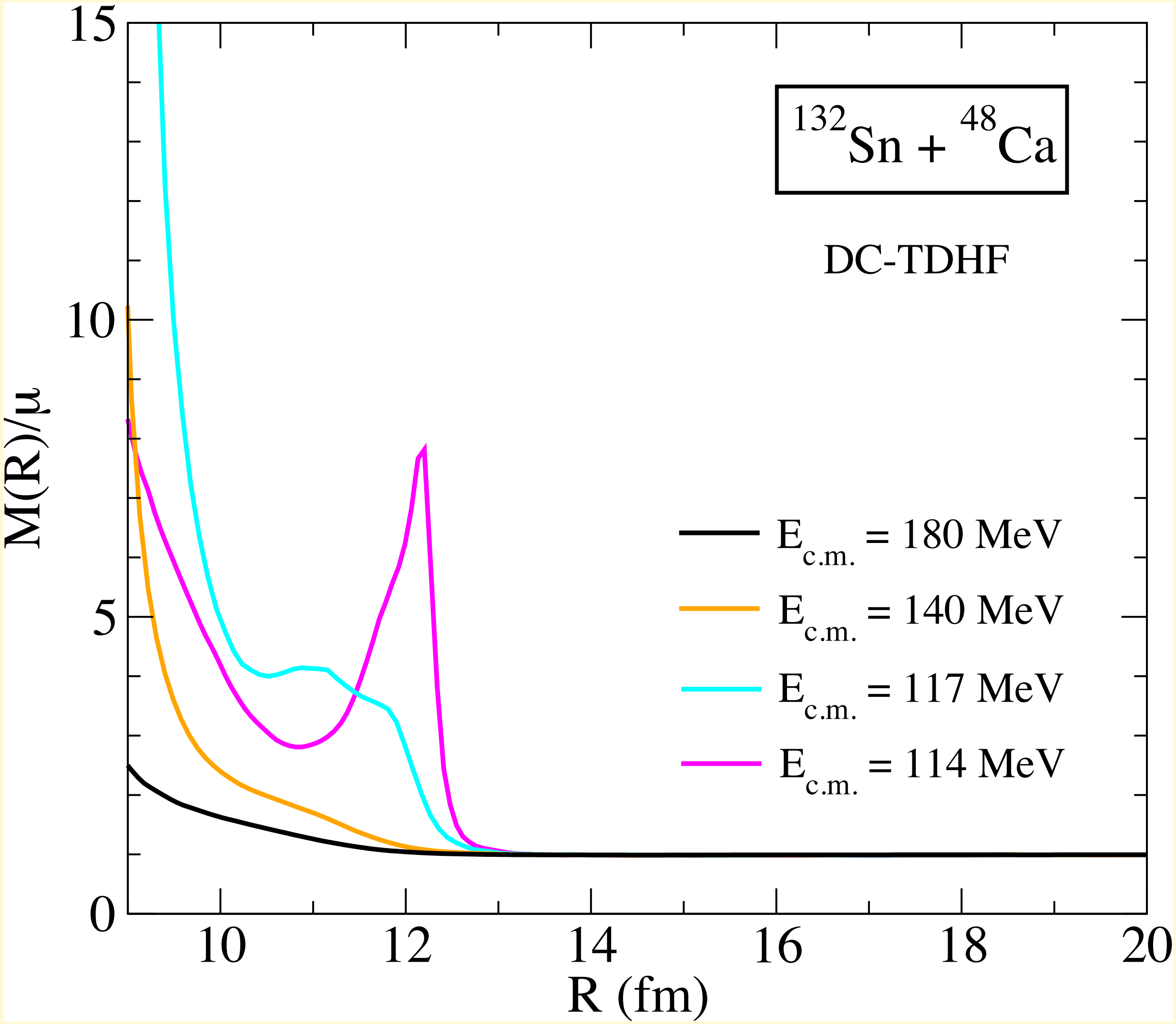
TDHF

DC-TDHF

$$M(R) = \frac{2\left[E_{\mathrm{c.m.}} - V(R)\right]}{\dot{R}^2}$$

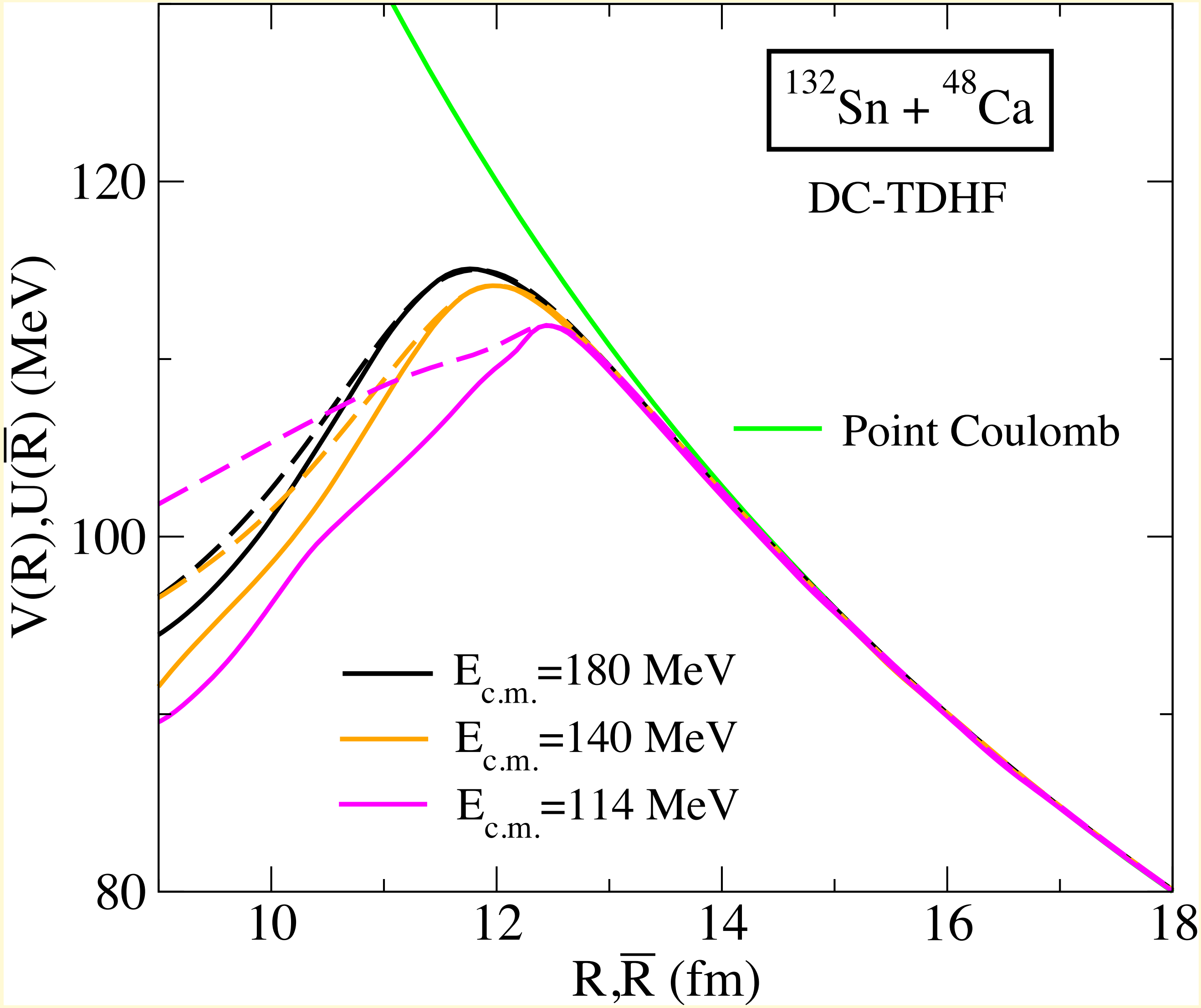
Mass parameter: strong E_{cm}-dependence

Oberacker, Umar, Maruhn, and Reinhard, Phys. Rev. C 85, 034609 (2012)



Transformed heavy-ion potential

Oberacker, Umar, Maruhn, and Reinhard, Phys. Rev. C 85, 034609 (2012)



Fusion / capture cross section for two spherical nuclei

$$\sigma_f(E_{\rm c.m.}) = \frac{\pi\hbar^2}{2\mu E_{\rm c.m.}} \sum_{\ell=0}^{\infty} (2\ell+1) T_\ell(E_{\rm c.m.})$$

total fusion cross section

Schrödinger equation for transformed radial coordinate

$$\left[\frac{-\hbar^2}{2\mu} \frac{d^2}{d\bar{R}^2} + \frac{\hbar^2 \ell(\ell+1)}{2\mu\bar{R}^2} + U(\bar{R}) - E_{c.m.} \right] \phi_\ell(\bar{R}) = 0$$

↑ reduced mass

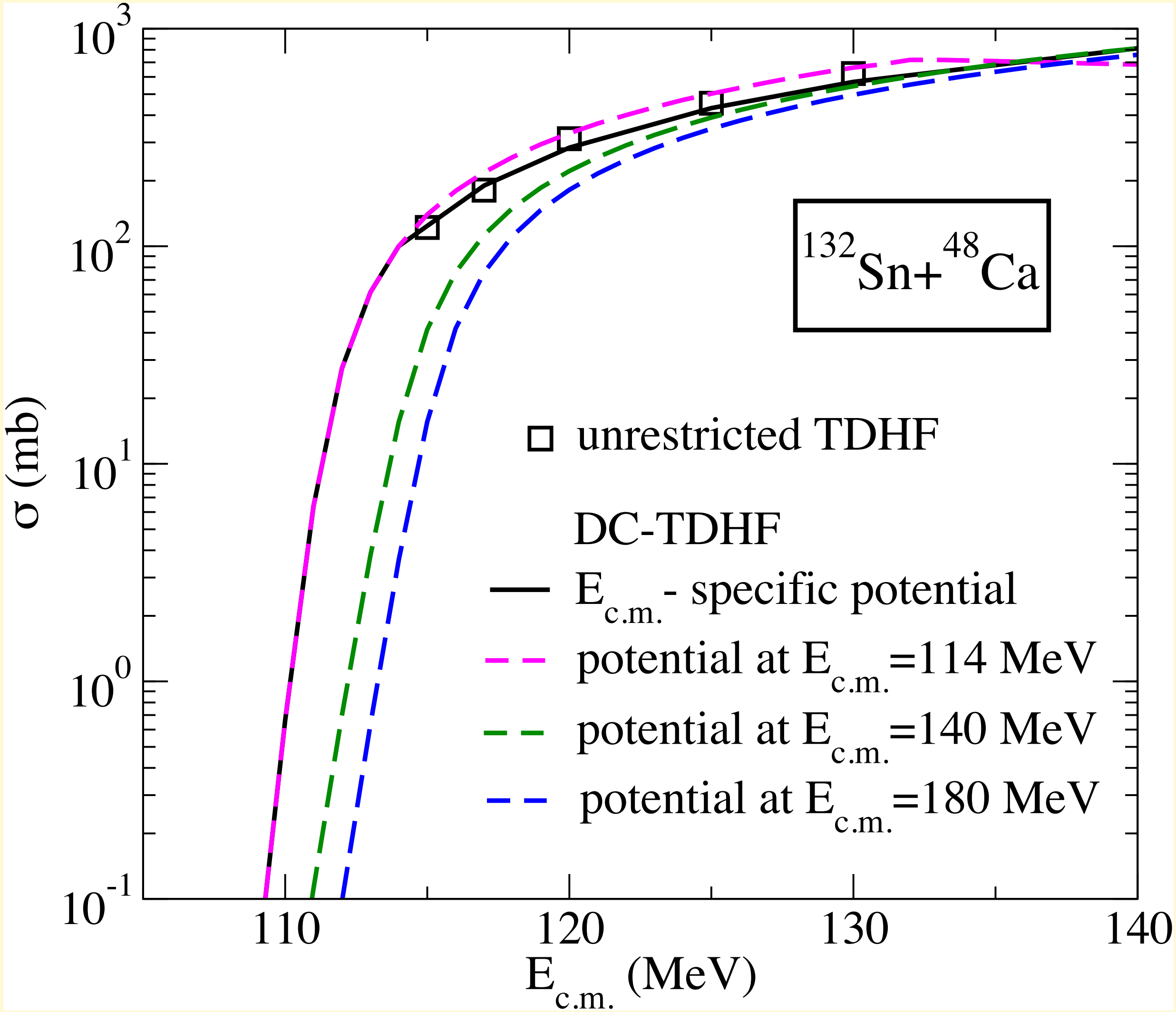
↑ transformed potential

solve Schrödinger equation numerically, with Incoming Wave Boundary Condition (IWBC)

$$\rightarrow T_\ell(E_{\rm c.m.})$$

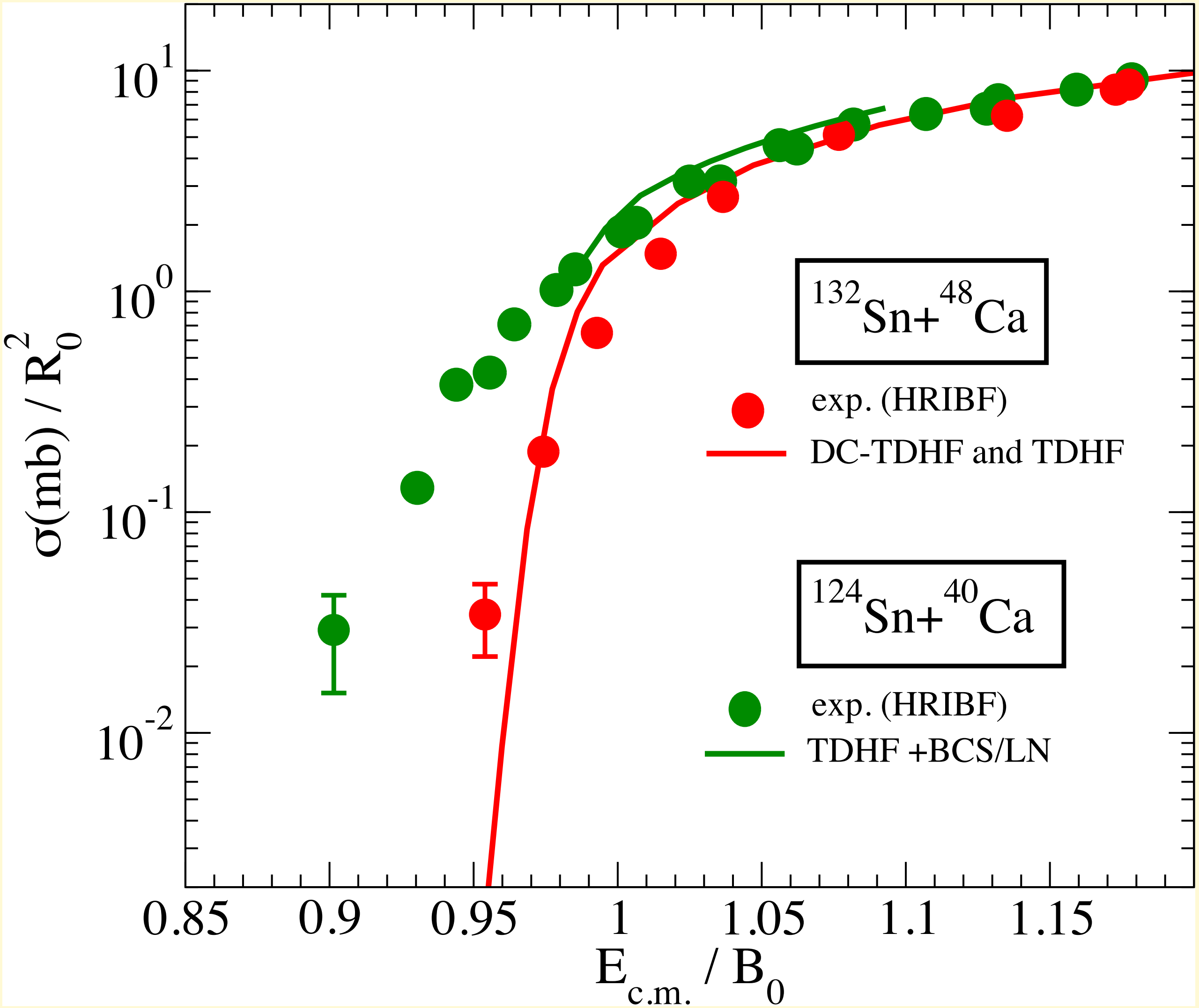
Total fusion cross section: TDHF and DC-TDHF

Oberacker, Umar, Maruhn, and Reinhard, Phys. Rev. C 85, 034609 (2012)



Fusion cross sections: theory vs. experiment

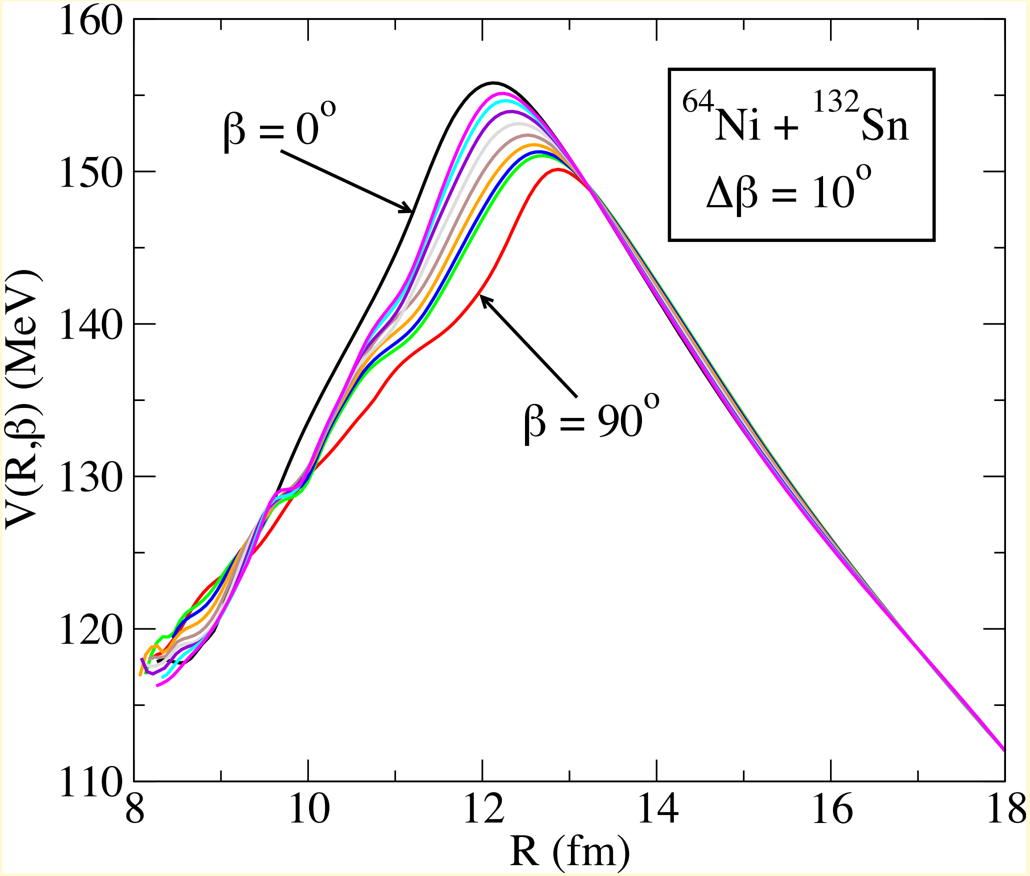
Scaling parameters: $R_0 = A_1^{1/3} + A_2^{1/3}$ $B_0 = Z_1 Z_2 / R_0$

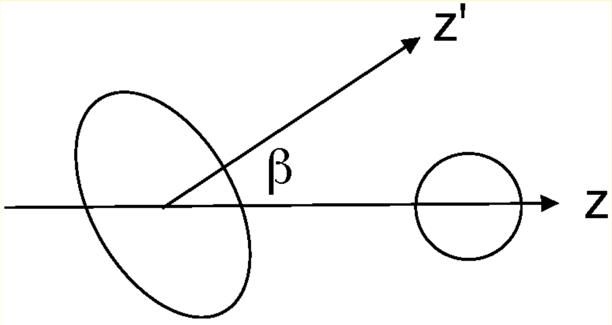


Oberacker, Umar, Maruhn, and Reinhard, Phys. Rev. C 85, 034609 (2012)

^{64}Ni + ^{132}Sn: heavy-ion interaction potential for deformed + spherical nuclei

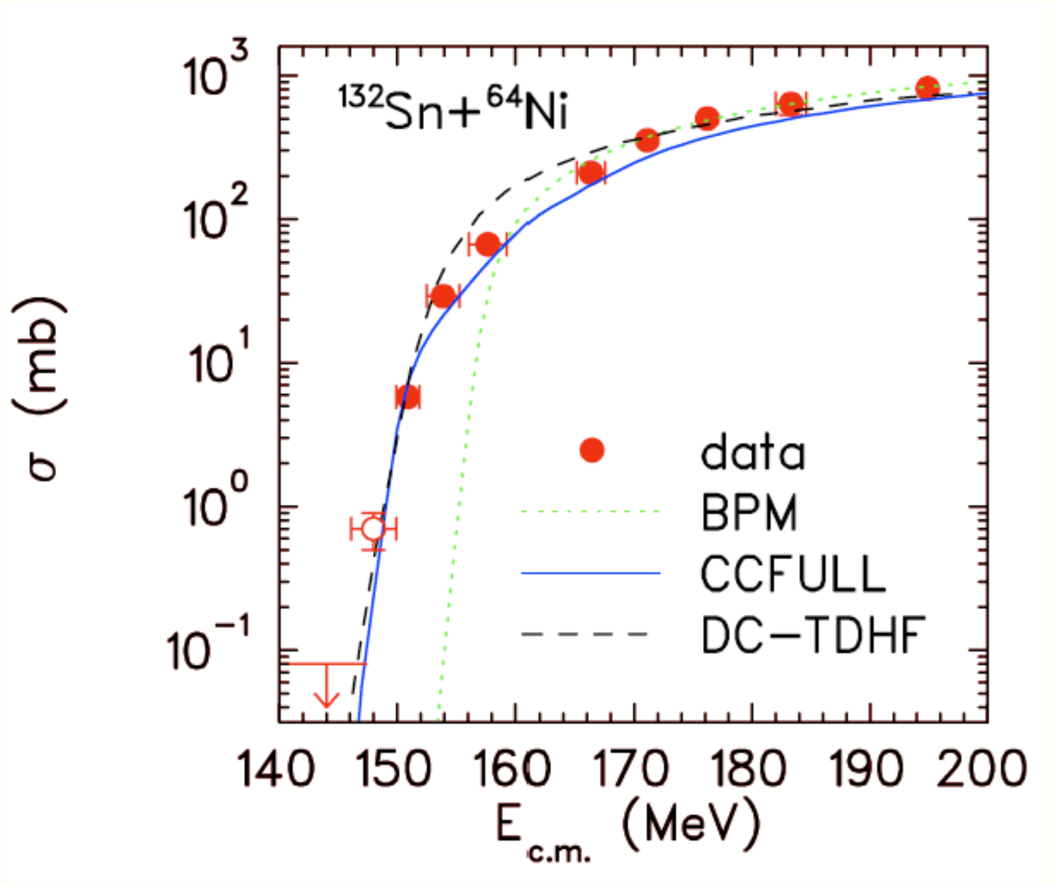
Umar and Oberacker, Phys. Rev. C 76, 014614 (2007)





Heavy-ion potential depends on initial orientation angle β of deformed nucleus

$^{64}Ni + ^{132}Sn$ Fusion Cross-Section



DC-TDHF theory

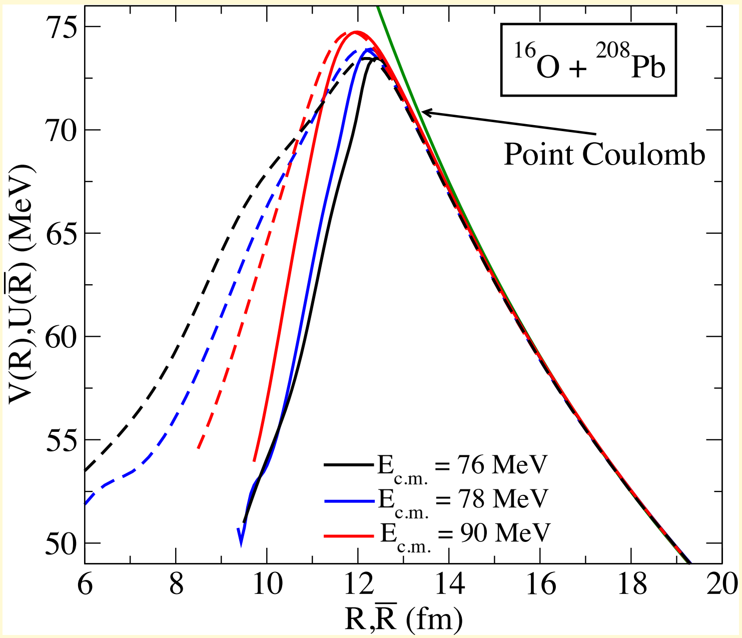
Umar and Oberacker,
PRC 76, 014614 (2007)

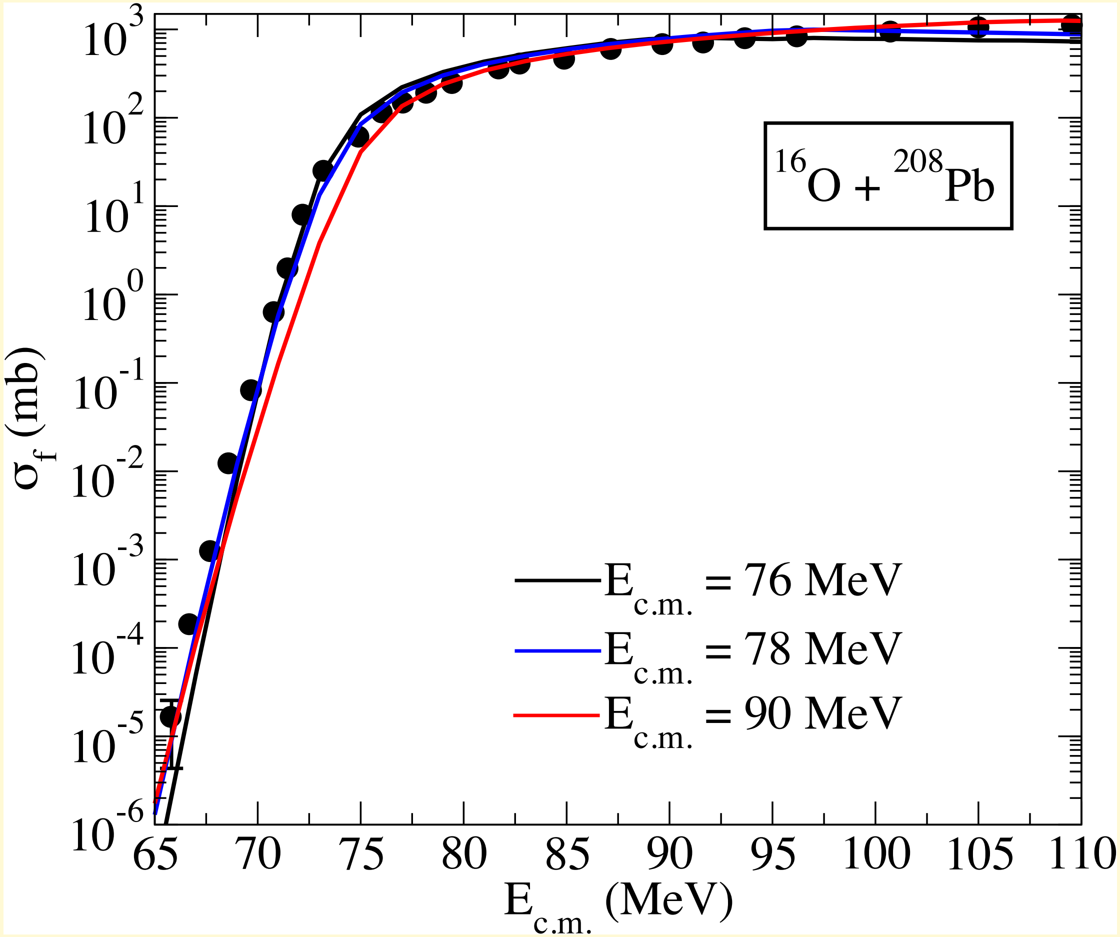
Exp. Data (HRIBF, ORNL)

J.F. Liang et al.,
PRL 91, 152701 (2003)
PRC 75, 054607 (2007)
PRC 78, 047601 (2008)

$^{16}O + ^{208}Pb$ fusion

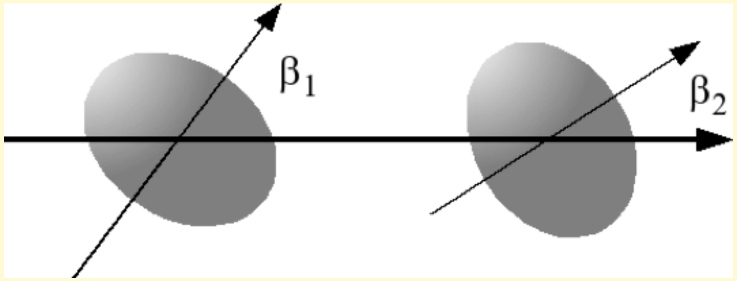
Umar and Oberacker, Eur. Phys. J. A 39, 243 (2009)

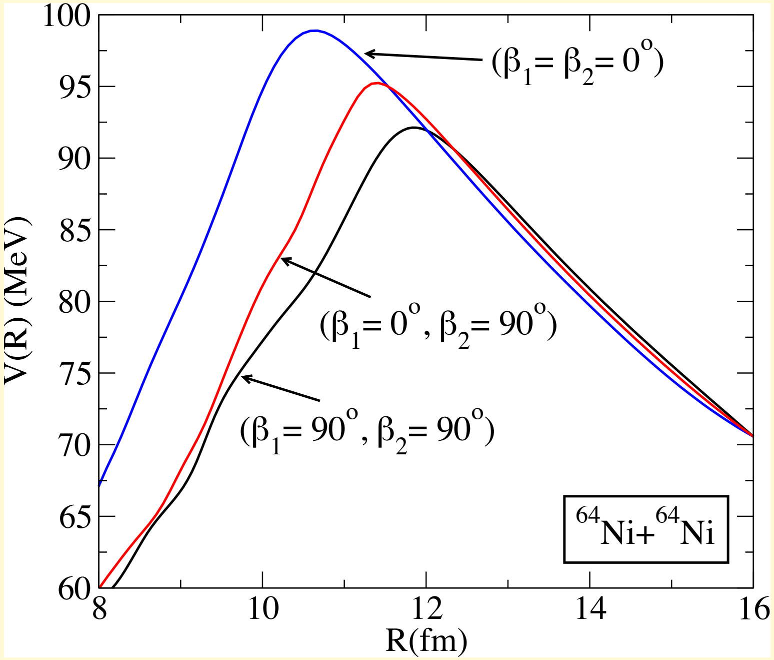


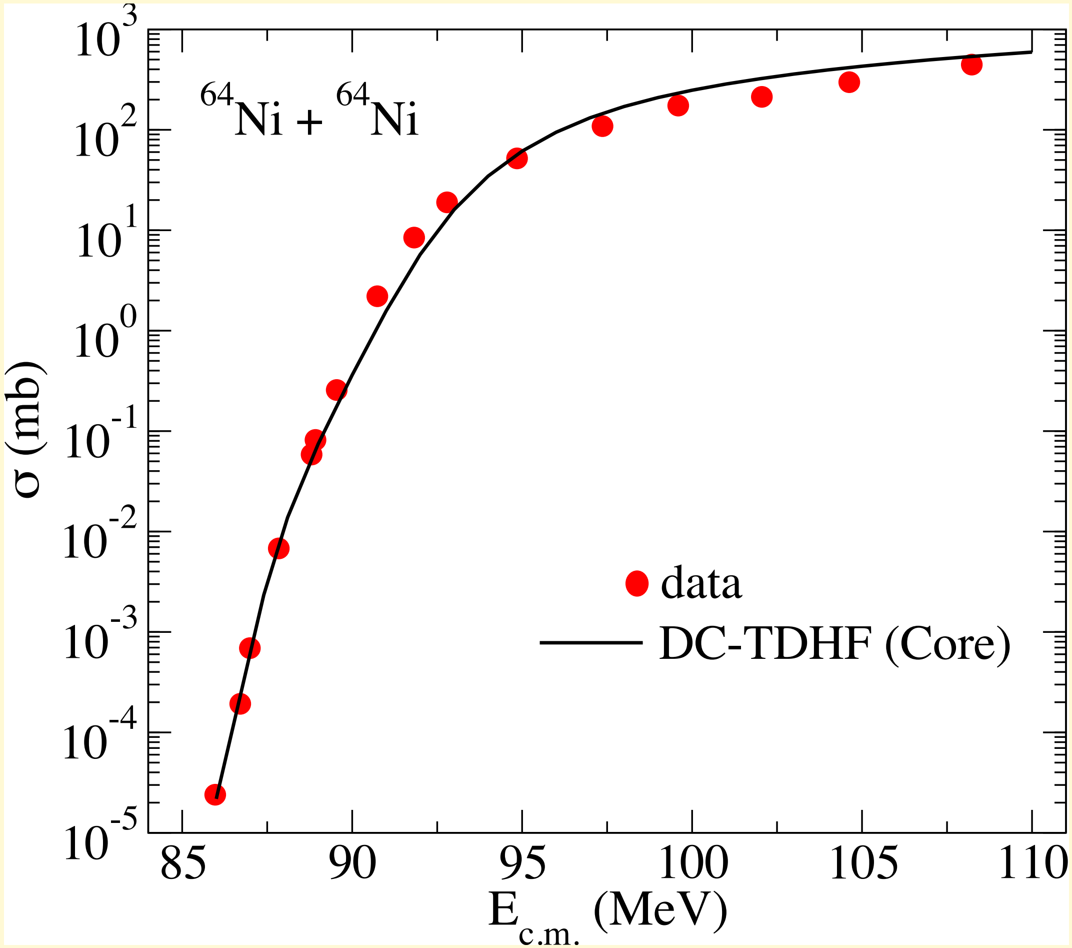


^{64}Ni + ^{64}Ni fusion

Umar and Oberacker, Phys. Rev. C 77, 064605 (2008)



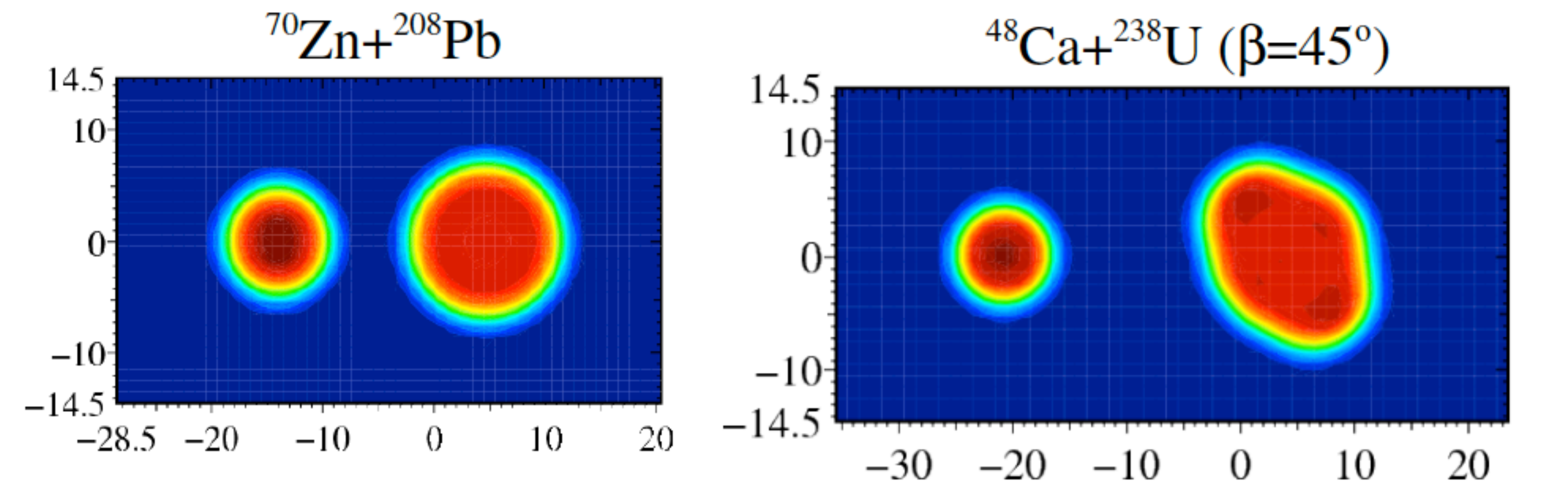




Heavy-ion fusion leading to superheavy element Z=112

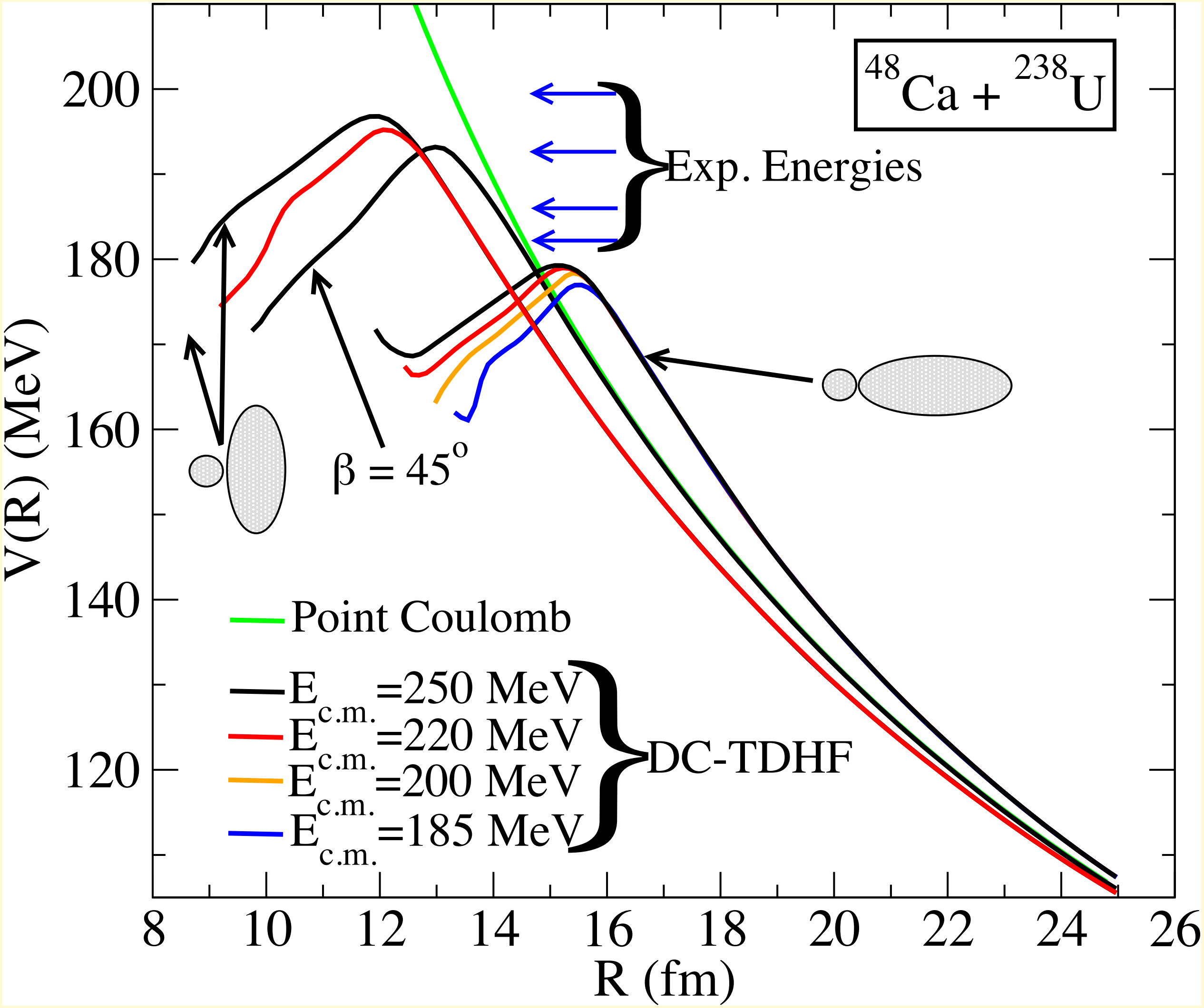
spherical + spherical:
"cold fusion" (E* small)

spherical + deformed:
"hot fusion" (E* large)



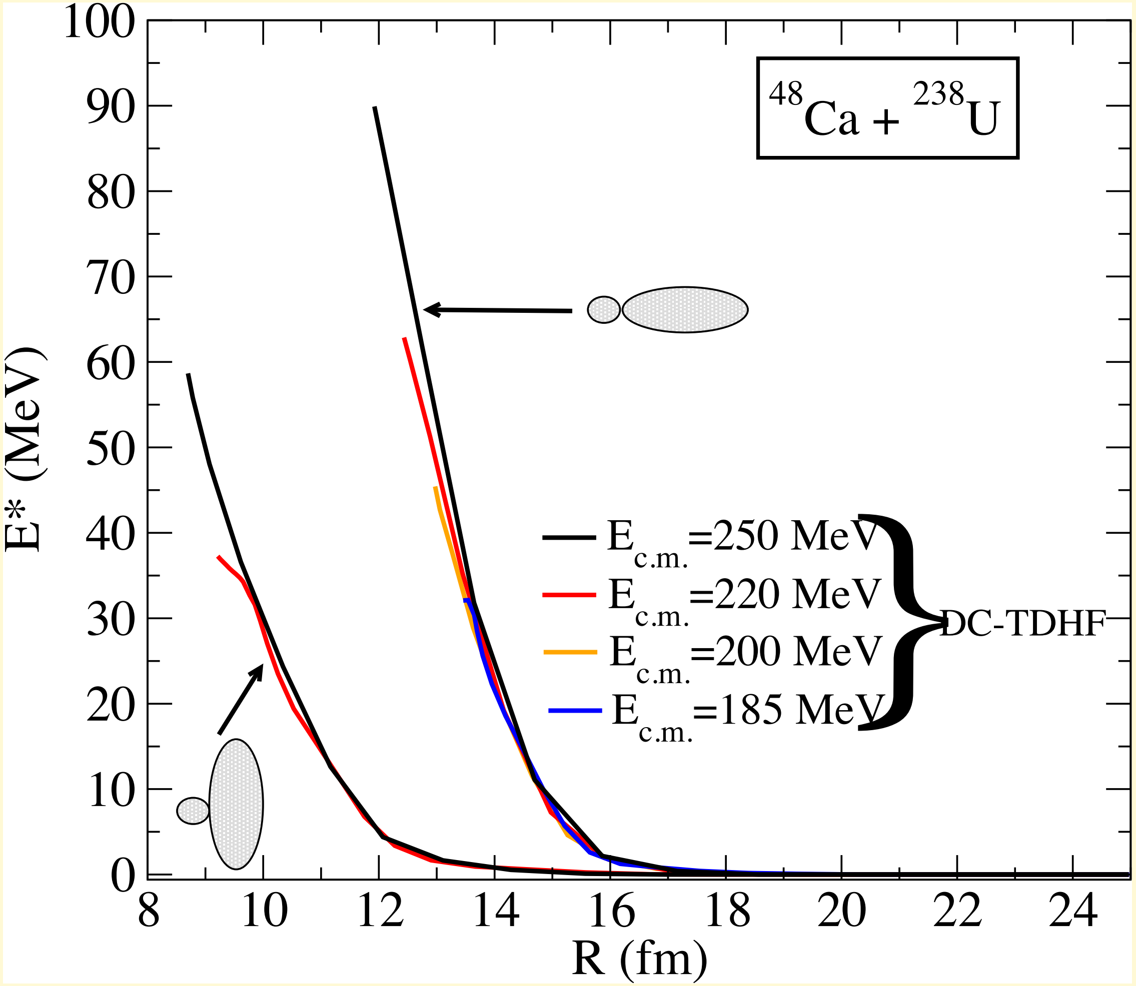
$^{48}Ca + ^{238}U$ heavy-ion potential

Umar, Oberacker, Maruhn & Reinhard, Phys. Rev. C 81, 064607 (2010)



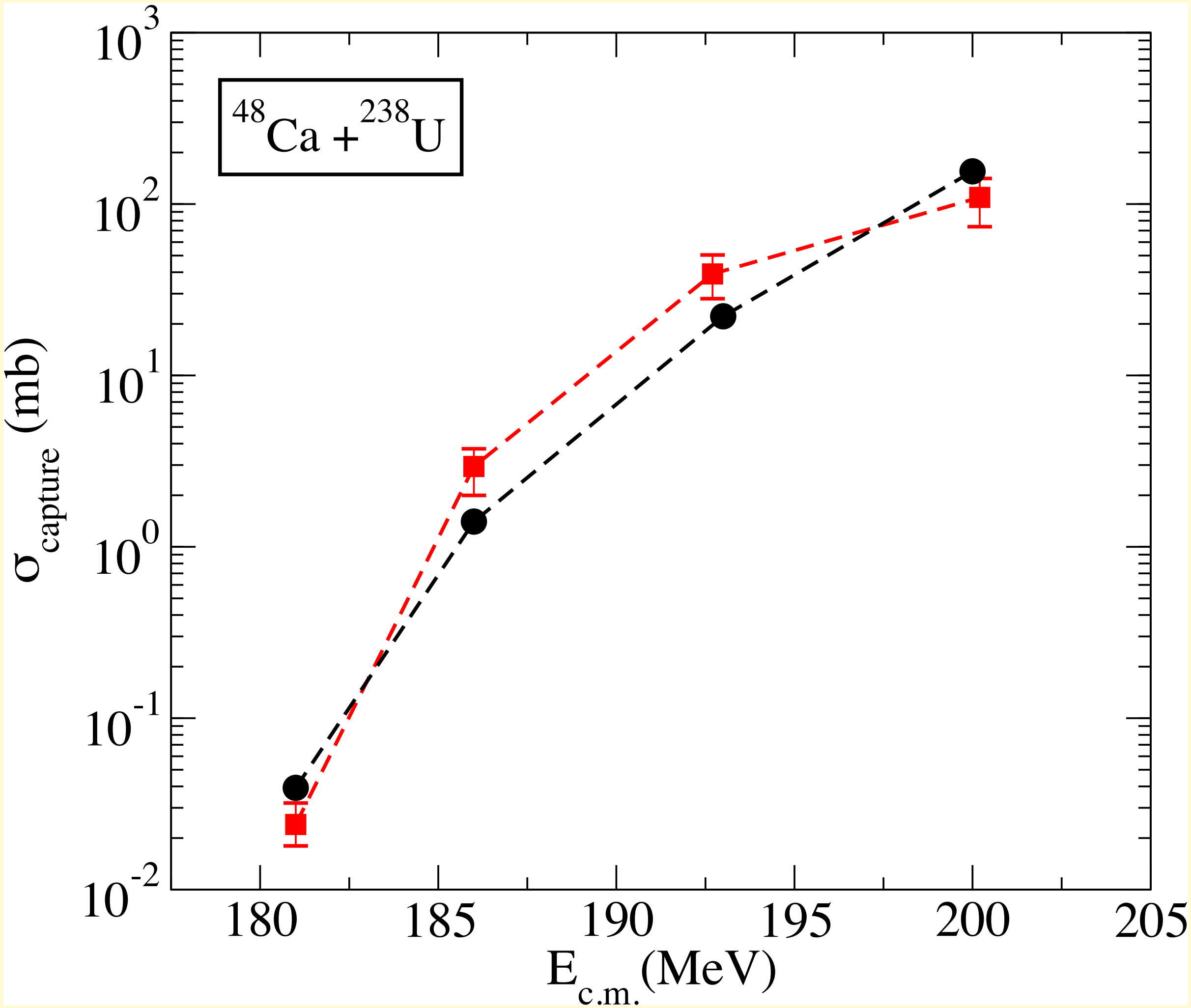
$^{48}Ca + ^{238}U$: excitation energy

Umar, Oberacker, Maruhn & Reinhard, Phys. Rev. C 81, 064607 (2010)



$^{48}Ca + ^{238}U$ capture cross section

Umar, Oberacker, Maruhn & Reinhard, Phys. Rev. C 81, 064607 (2010)

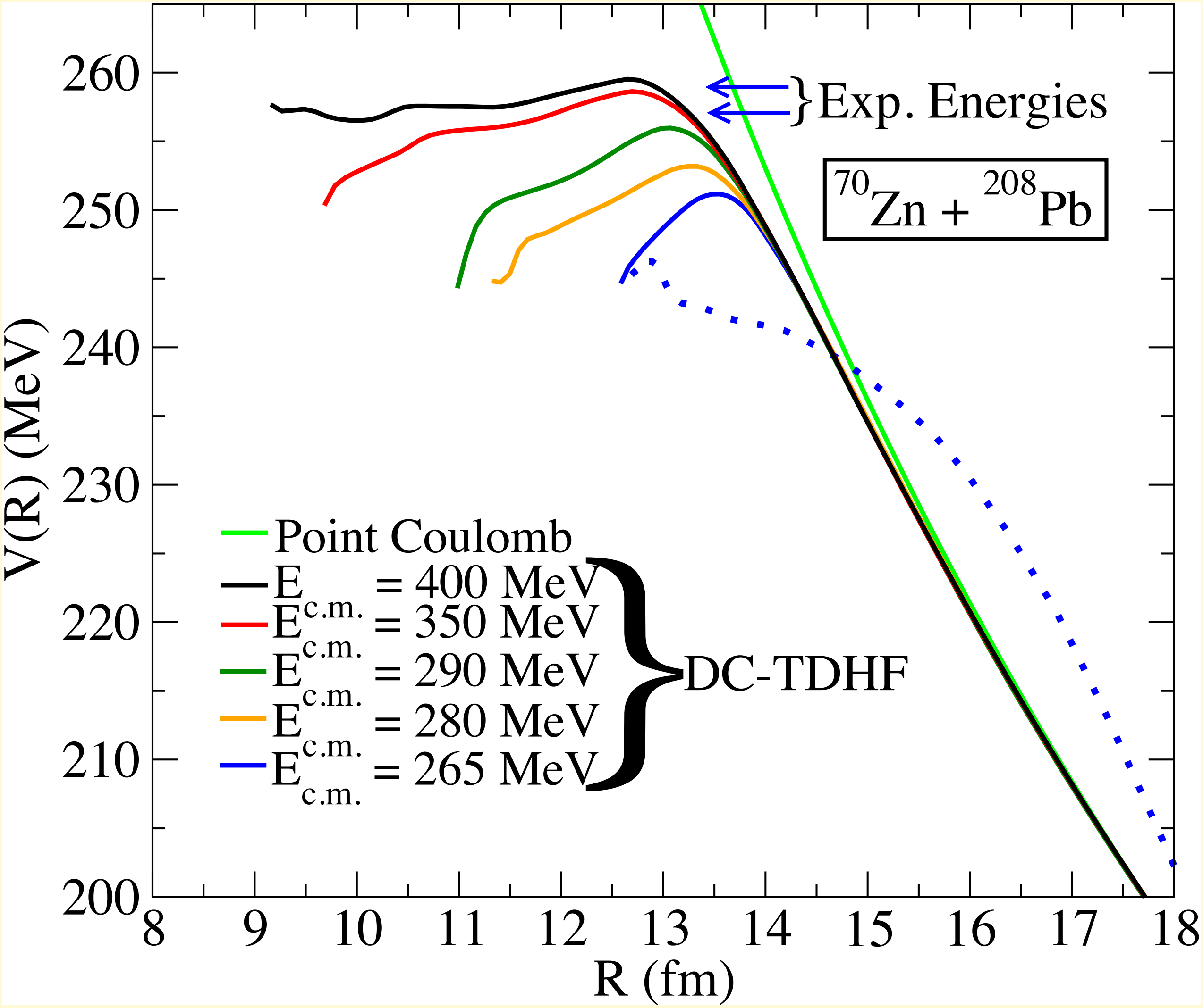


● DC-TDHF

■ Experiment: Oganessian *et al.*, J. Phys. G 34 R165 (2007)

$^{70}Zn + ^{208}Pb$ heavy-ion potential

Umar and Oberacker, Phys. Rev. C 76, 014614 (2007)



Excitation energies at capture point in hot and cold fusion

Umar and Oberacker, Phys. Rev. C 76, 014614 (2007)